INFORMATION, INSIDER TRADING, EXECUTIVE RELOAD STOCK OPTIONS, INCENTIVES, AND REGULATION

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Abstract. We introduce a theoretical model of executives with insider information who receive executive stock options (ESOs) as incentives and optimize their "outside wealth" portfolios. We show that insider information nullifies ESO incentivizing, misaligning executives' and shareholders' interests. We offer realigning methods: granting executives with reload stock options (RSOs) while imposing a blackout trading period. Effective blackouts keep executives incentivized without over-restricting, i.e., reducing executives' welfare below that of outsiders. We introduce RSO pricing for insider executives and offer policy implications: reestablishing the currently "out of favor" RSOs, allowing firms, not regulators, to set effective blackouts on securities they issue.

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1 Introduction

We introduce a theoretical model in which executives, privy to insider information that improves predictability of future returns, optimize their personal "outside wealth" portfolios while being incentivized by conventional non-transferable non-hedgeable executive stock options (ESOs).[1] Future events that insiders have information about could be announcements regarding earnings, mergers or acquisitions, capital structure, or research and development outcomes. We define "outside wealth" as executives' personal wealth, not including nonvested stocks and options the firm gives them as (incentivizing) compensation.

We demonstrate that, while trading their outside wealth, executives' use of their insider information, even noisy information, on stock return shocks occurring at a known future time generates infinite derived utility.[2] This utility effectively nullifies conventional ESO incentives, consequently nullifying the alignment of executives and stockholder interests.[3]

We show that blackout trading periods (blackouts)[4] imposed on executives reduce the benefits of their insider information and may restore the conventional ESO incentivizing mechanism, if they are set effectively. We identify effective blackouts by lower and upper duration bounds. Lower bounds are the shortest blackout under which executives do not choose to exercise all their ESOs once vested, as immediate exercise eliminates their alignment of interests with those of shareholders. Upper bounds are the longest blackout at which executives' derived utility under insider information is equal to outsiders' derived utility with the same initial total wealth but no blackout. Thus, lower bounds define the minimal blackout lengths required for keeping executives incentivized, and upper bounds define the longest blackout trading period beyond which executives are worse off than outsiders. We call periods unfair when they extend beyond the upper bounds.

We note that idiosyncratic insider information requires blackouts that apply to a firm's stock only, whereas insider information with a systematic component might require blackouts that apply to all stocks.

However, imposition of a blackout trading period, by itself, is not a satisfactory solution for several reasons. First, an effective blackout might not always exist.[5] Second, effective blackout duration depends on executives' four specific attributes: their outside wealth level and its composition, and their

[1] ESOs are non-transferable non-hedgeable American stock options.
[2] For examples in which expected utility is not infinite, see, e.g., Pikovsky and Karatzas (1996), Grorud and Pontier (1998, 1999), Corcuera et al. (2004), Hillairet and Jiao (2017), or D'Auria and Salmerón (2020). Expected utility can also be finite if one adds constraints on trading, such as constraints on borrowing or short sales. See, e.g., Pikovsky and Karatzas (1996) Section 5.
[3] Infinite derived utility requires insider executives to rebalance their portfolio at infinitely high frequency as the holding period approaches the public announcement of the insider information they have. Because continuous rebalancing is not realistic, the statement can be understood as sufficiently high derived utility from outside wealth using insider information makes executives lose their interest in increasing their wealth from incentives.
[4] A blackout begins a certain period prior to the public announcement of an event and ends upon completion of one full trading day after the announcement.
[5] Mathematically, blackout lower bounds occur on earlier calendar dates than their upper bounds.

insider information type[6] and its precision. Third, even for a particular executive, the required duration to make a blackout effective dynamically changes.[7] Fourth, job termination dates have an overriding effect on effective blackout duration. Finally, effectiveness of a blackout depends on ESO allocations. A stronger incentive, allocating additional ESOs, might render the blackout trading period too short, inducing exercise of all incentivizing ESOs and resulting in a total liquidation of the proceeds and, consequently, causing executives to lose their sensitivity to the incentivizing effects. We call this the *ESO tolerance effect*. Thus, imposing a blackout cannot, by itself, resolve the incentivizing failure of conventional ESO. The intuition driving this result is simple. As it is impossible to maintain an effective blackout, executives find it optimal, sooner rather than later, to exercise their ESOs, nullifying the ESO incentives.

We identify a mechanism that restores executives' incentives regardless of whether they have insider information: a combination of granting non-transferable non-hedgeable executive reload stock options (RSOs) with infinite reload and imposing a blackout. RSOs are ESOs that upon exercise (i.e., reload) are paid for by using the underlying stock (rather than cash) and converted to new at-the-money ESOs. RSOs with infinite reload allow an unlimited number of reloads. We show that optimal RSO exercise is at the times when the underlying stock price hits a new high and that this result holds when option holders have insider information at any informational quality level and when RSOs are non-transferable non-hedgeable.

We introduce analytical (subjective and objective) pricing of non-transferable non-hedgeable RSOs with infinite reloads for insider executives. We merge two analytical approaches. The first approach is the constrained primary assets portfolios optimization techniques of Cvitanić and Karatzas (1992) and Karatzas and Kou (1996), which Colwell, Feldman and Wu (2015) developed to price conventional non-transferable non-hedgeable American ESOs. The second approach is the enlarged filtration technique. Some of the earliest publications in this area are from Jeulin and Yor (1978), Yor (1978) and Jeulin and Yor (eds) (1985), which includes papers by Chaleyet-Maurel and Jeulin, Jacod, Jeulin and Yor, and others. See also, Al Hussaini and Elliott (1987). Some of the earliest papers on insider trading in finance are due to Kyle (1985) and Back (1992), but the first papers to rigorously use the enlargement of filtrations approach include Pikovsky and Karatzas (1996), Elliott, Geman, and Korkie (1997), Amdinger et al. (1998), and Baudoin (2003). We first learned of this area of research via the paper by Pikovsky and Karatzas (1996), so much of our notation is due to their paper. Using this approach, we introduce a method of enlarging the filtration that could be thought of as noisy insider information, a model that does not seem to appear in the literature, although many models of insider

[6] The type of insider information could be the terminal value of the risk source (equivalently the increments from the current value, which is the focus of this study), the upper bound at a fixed time, the local time at a fixed time, the last zero before a fixed time, a first hitting time, etc., or a combination of these. See Mansuy and Yor (2006, p. 34).
[7] Effective blackout lower and upper bounds are both functions of executives' total wealth levels and ratios of nonvested compensation values to total wealth, which are dynamic.

information exist. For papers that deal with *weak information*, in which the additional information available to insiders is about the distribution of the asset's returns rather than the actual value of the asset's returns; see, e.g., Baudoin and Nguyen-Ngoc (2004); for a model using the *progressive enlargement of filtrations*, in which the additional information involves knowledge about a stopping time that is not accessible to an uninformed trader, see, e.g., Imkeller (2002). For asymmetric information in models with jumps, see Elliott and Jeanblanc (1998) and Grorud (2000); for insider information as applied to credit derivatives, see Hillairet and Jiao (2011); and for insider trading with a more general formulation of the enlarged filtration, see Biagini and Øksendal (2005). Leon et al. (2003) take a Malliavin calculus approach to maximizing the expected utility of an insider under logarithmic utility. Amendinger et al. (2003) evaluate the monetary benefit of insider knowledge rather than calculating the expected utility gain for inside traders. Grorud (2001) discusses market completeness and arbitrage opportunities in a market with an informed trader. See also Hillairet (2005) and the books by Hillairet and Jiao (2017) and Aksamit and Jeanblanc (2017).

Our results shed new light on the value and role of RSOs. Our ability to analytically price non-transferable non-hedgeable RSOs with infinite reloads for insider executives should put an end to calling RSOs "money pumps," and should change the Financial Accounting Standards Board's attitude toward RSOs. The Board responded to past difficulty of pricing RSOs by requiring firms granting RSOs to account for the RSOs as a separate award [see FAS123(R) paragraph 24 to 26 and Saly et al. 1999]. Our results might deem this requirement unnecessary.

We run Monte Carlo simulations of six scenarios: executives with good/bad news insider information, and outsider executives, repeated in high/low volatility regimes. Our main findings include these: (i) subjective prices perceived by insider executives are usually greater than firms' granting costs, but when executives have insider information, ESO incentives could become weaker. Therefore, the overall granting efficiency (definition in Section 6) of ESOs to insider executives, and more so in low-volatility regimes could be low; (ii) high-volatility regimes with good (bad) news information, induce ESO granting deficiency (efficiency) most of the time. The simulation sensitivity analysis is consistent with our theoretical results.

Our results are mostly consistent with, and provide theoretical foundations for, the following empirical research progression. Roulstone (2003) found that insider trading laws increase executive compensation and share-based incentives. Denis and Xu (2013) showed that those results are robust to alternative definitions of insider trading restrictions and enforcement and to panel regressions with country-fixed effects. Henderson (2010) further studied the same relationship but focused on Rule 10b5-1 and isolated the potential profits from portfolio optimization and informed trading. The evidence suggests that executives whose trading freedom increased using Rule 10b5-1 trading plans experienced reductions in other forms of pay to offset the potential gains from trading. Carpenter and Remmers (2001) and Aboody et al. (2008) examined whether insiders use private information to time the exercise of their ESO. Fu and Ligon (2010) investigated whether insider information motivates executives' early exercise upon vesting. Bettis et al. (2005), taking executives' insider role into account, calibrated the Carpenter (1998) utility-based model creatively to get ESO values and incentives, and documented the impact on insiders' exercise behavior. Brooks et al. (2010) found that the best-informed executives tended to exercise early, and the operating performance of firms following exercises motivated by private information was significantly worse than that of firms in which the exercises were not motivated by private information.

There are several policy implications. First, our ability to price executive RSOs and to demonstrate RSOs' essential role in executives' incentivizing suggest that it might have been premature for RSOs to fall out of favor. Perceiving RSOs as a "money pump" seems to have been a misunderstanding, and FABS's concerns regarding RSO pricing difficulty are now mitigated. Second, we demonstrate how careful use of a blackout is important for maintaining insider executives' incentives on the one hand and fairness on the other. Third, we demonstrate how the combined use of RSOs and blackouts is essential to executives' incentives and, thus, to effective corporate governance. Finally, because of the conditional nature of effective blackouts and firms' superior personal information about their executives relative to the Securities Exchange Commission (SEC), the firms should determine the effective blackout for their executives and the SEC should regulate the blackout trading prohibition. Indeed, it is the SEC's regulatory capacity to restrict executives from non-premeditated trading in their firm's stocks if insider information is idiosyncratic. If executives' information is systematic, the SEC may extend the restrictions to retirement funds and other outside wealth components.

Section 2 reviews legal essentials. Section 3 models insiders' constrained portfolio optimization. Sections 4 and 5 make policy recommendations on understanding blackout trading period regulation and designing efficient firm incentives, respectively. Section 6 discusses simulation results. Section 7 concludes.

2 Legal essentials

Executives may use insider information in two trading styles: arbitrage style, with which they profit using long and short positions to approach zero net investment and zero market exposure, and portfolio optimization using insider information to improve portfolio processes.

Although legal obligations are not conditional on trading styles, insider arbitrage was effectively made illegal by anti-fraud decrees of the Securities Exchange Commission [see Section 10(b) and Rule 10b5 of the Securities Exchange Act of 1934, Trading Sanctions Act of 1984 (ITSA); see also Bainbridge 2013] because this style of trading is relatively easy to detect because of a large trading volume.

In the case of portfolio optimization, according to Rule 10b5-1 of the Act, insiders may use premeditated portfolio optimization style trading plans to avoid accusation. Unlike arbitrage, distinguishing between insiders' and outsiders' portfolio processes is very difficult until optimal trading for insiders becomes extremely large near announcement time. Moreover, it is generally impossible to identify insider information arrival times and quality[8]; hence, being uninformed and being informed with very noisy information are not technically distinguishable. However, we will show in Section 4.1 that attaining infinite utility with noisy information is possible as long as it is not pure noise. Therefore, insiders can take advantage of Rule 10b5-1, which allows trading plans initiated before insider information arrivals, because it is difficult to enforce.

Also, although insider trading facilitates rapid price discovery and enhances market informational efficiency, the price change caused by insider arbitrage is a one-off instant occurrence per information shock, and insiders are the only beneficiaries. In contrast, insider portfolio optimization is a sustained information release process, and there is time for profit sharing among insiders and outsiders. We can see this situation as a rationale for Rule 10b5, which prohibits insider arbitrage, and Rule 10b5-1, which allows insiders to execute premeditated trading plans.[9]

We focus in this study on insiders' portfolio optimization style trading. Specifically, we adopt an approach similar to Pikovsky and Karatzas (1996) in which insiders have some information, unavailable to most traders, about the stock spot price that will prevail at a future time T^*. When they use this information, their optimal portfolio process yields higher returns. As the holding period approaches T^*, derived utility gains, due to their insider information, sufficiently increase to overwhelm the utility derived from their ESO, rendering the ESO and its incentivizing irrelevant.

Thus, an adequate blackout trading period is required. Section 306(a) and Regulation Blackout Trading Restriction under the Sarbanes-Oxley Act of 2002 prohibit corporate executives and rank-and-file employees from engaging in transactions during a blackout in order to invalidate the necessary condition for insiders to achieve infinite derived utility—continuous trading in the neighborhood of T^*. For example, a blackout might be enacted to begin two weeks prior to the end of a fiscal quarter and end upon completion of one full trading day after the public announcement of earnings for that quarter.

Regarding the insider trading restriction on exercising options, exercising through an "intra-company" approach, i.e., executives providing value to the company in the form of cash or shares in exchange for more shares, is not a violation of Rule 10b5. Any other approach involving contemporaneous sales into the market is prohibited, e.g., the "broker-assisted cashless" exercise whereby, at the time of exercise, some or all the exercised shares are sold into the market, the requisite amount of the sale proceeds is used to pay the company for the exercise, and the holder keeps the net proceeds and any unsold shares. Even when taking the "intra-company" approach, the executives cannot sell the resulting shares during the blackout period (See Nathan and Hoffman 2013).

Furthermore, Statement of Financial Accounting Standards No. 123 (revised 2004) [FAS123(R)], paragraph B69 to B72 (B80 to B82) precludes executives from transferring (hedging)

[8] Grorud and Pontier (1999) identified insider trading by constructing a statistical test to compare insiders' and outsiders' trading strategies. See also Grorud and Pontier (1998).

[9] Insiders must initially file with the SEC Form 3 stating ownership of firm securities, must report ownership changes on Form 4, and deferred such reporting on Form 5 so that outsiders can benefit from the information disclosed.

nonvested share-based compensations, including both nonvested share options and nonvested shares, to third parties, i.e., their options are non-transferable non-hedgeable. Also forbidden is a "short-equivalent position", i.e., adopting trading strategies whose net replicating position on the firm's stock is a short position.

Therefore, the insider trading liability on a blackout trading period, the intra-company exercise,[10] and the non-transferable non-hedgeable rules together form insider executives' portfolio constraints.

3 The model

Our model is designed to provide a legal basis for promulgating a blackout trading period. In this section, we show that, if a blackout is not applied, insider portfolio optimization invalidates executives' incentives. We focus on an executive who optimally trades until time $T \in [0, T^*]$, has noisy information of the firm's stock value at T^*, holds n non-transferable non-hedgeable ESOs, and trades outside wealth optimally in a market with one risk-free asset and d primary assets.

3.1 Noisy information

We assume executives observe a mixture of true information and noise over time rather than accurately knowing in advance the terminal value of a risk source. We model noise as an additional risk source associated with an imaginary primary asset that is non-tradable.

In particular, we describe the original market, $\mathcal{M}$, in which there is a traded bond whose price evolves according the differential equation

$$dS_0(t) = S_0(t)r(t)dt, \;\; S_0(0) = 1, \tag{1}$$

where $r(t)$ is a scalar interest rate.

The uncertainty is driven by a $d+1$ dimensional standard Brownian motion, $W = (W_1, W_2, \dots, W_{d+1})^\top$, in $\Re^{d+1}$, defined on a complete probability space on $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$, where $\{\mathcal{F}_t\}$ is the $\mathbb{P}$-augmentation of the natural filtration $\mathcal{F}_t^W = \sigma(W(s); 0 \le s \le t)$, with fixed time span $[0, T^*]$ for some finite $T^* > 0$.

The primary asset prices $S_i(t), i = 1, \dots, d$ follow the dynamics of

$$dS_i(t) = S_i(t)\left[b_i(t)dt + \textstyle\sum_{j=1}^{i} \sigma_{i,j}(t)dW_j(t)\right], \;\; S_i(0) = s_i, i = 1, \dots, d. \tag{2}$$

Without loss of generality, we assume that $S_1(t)$ is firm stock price.

The price of an imaginary asset that serves as noise follows the dynamics

$$dS_{d+1}(t) = S_{d+1}(t)[dW_{d+1}(t)], \;\; S_{d+1}(0) = s_{d+1}; \tag{3}$$

that is, without loss of generality, we can set $b_{d+1}(t) = 0$ and $\sigma_{d+1,d+1}(t) = 1$. Here $\sigma(t) \triangleq \left(\sigma_{i,j}(t)\right)_{1 \le i,j \le d+1}$ is a volatility matrix. The submatrix for $i, j \le d$ is the lower unit triangular of the

[10] The intra-company approach requires executives to keep cash or shares for payment upon exercise. This imposes portfolio constraints on insider executives additional to blackout and non-transferable non-hedgeable.

Cholesky decomposition of the positive definite variance-covariance matrix of the primary asset's return vector $(dS_i(t)/S_i(t)), i \leq d$, and $b(t) \triangleq \left(b_1(t), \dots, b_{d+1}(t)\right)^\top$ is a drift rate vector. We assume that $r(t)$, $\sigma(t)$ and $b(t)$ are progressively measurable with respect to $\{\mathcal{F}_t\}$. The market price of risk is a process defined as

$$\theta(t) \equiv \sigma^{-1}(t)[b(t) - r(t)\mathbf{1}], \tag{4}$$

where $\mathbf{1} = (1, \dots, 1)^\top$, and we assume that $\mathbb{E}\int_0^{T^*} \|\theta(t)\|^2 dt < \infty$.

At time t, we assume that the executives can observe process $G(t)$, the mixture of true information $W_1(T^*) - W_1(t)$ and noise $W_{d+1}(T^*) - W_{d+1}(t)$ on top of the realized value of $W_1(t)$; i.e.,

$$G(t) \equiv W_1(t) + \lambda[W_1(T^*) - W_1(t)] + \sqrt{1-\lambda^2}[W_{d+1}(T^*) - W_{d+1}(t)], \tag{5}$$

with constant information quality coefficient $\lambda \in [0,1]$. A greater λ indicates a higher precision of insider information. Then, to insider executives, the complete probability space where Brownian motion W is defined on is $(\Omega, \mathcal{G}, \mathbb{P})$, the probability measure $\mathbb{P}$ is unchanged, the filtration is enlarged from $\mathcal{F}$ to $\mathcal{G} = \{\mathcal{G}_t\}_{0 \leq t \leq T^*}$ with $\mathcal{G}_t \equiv \mathcal{F}_t \vee \sigma\left(G(t)\right)$, and G_t is $\mathcal{F}_{T^*}$-measurable and $\mathcal{G}_t$-measurable.

Because

$$\widetilde{W}(t) = W(t) - \int_0^t a(s)ds$$

is a Brownian motion on $(\Omega, \mathcal{G}, \mathbb{P})$, where the compensating process $a(t)$ is uniquely determined by $dq_t^y / q_t^y = a(t)dW(t)$ and $q_t^y \equiv \frac{P(G \in dy|\mathcal{F}_t)}{dy}$, as we show in Corollary 2 in Section 3.2 below. Thus, we can apply Girsanov's Theorem on $(\Omega, \mathcal{G}, \mathbb{P})$ and state the following corollary.

Corollary 1. On $\mathcal{G}_t$, $\forall t \in [0, T^*)$, $\left.\frac{d\mathbb{Q}^{\mathbb{G}}}{d\mathbb{P}}\right|_{\mathcal{G}_t} = e^{-\int_0^t \theta_a(s)d\widetilde{W}(s) - \frac{1}{2}\int_0^t (\theta_a(s))^2 ds}$, with $\theta_a(t) = \theta(t) + a(t)$, is the Radon-Nikodym derivative changing the probability measure from $\mathbb{P}$ to $\mathbb{Q}^{\mathbb{G}}$, under which the discounted stock price $e^{-\int_0^t r(s)ds} S(t)$ is a martingale.

Our model changes the nature of insider information; it is a modification of Equation (3.1), Pikovsky and Karatzas (1996, p. 1103) and has three intuitively appealing properties. First, insiders continuously observe a stochastic noisy process of the risk source rather than one T^* – noisy signal. Second, the noisy insider information under this setting is unbiased, i.e., for all $t < T^*$, $\mathbb{E}^{\mathbb{P}}[G(t)|W(t)] = W_1(t)$. Third, the quality and precision of insider information increases proportionally to the inverse of the remaining time, up to the date at which the information is disclosed, i.e., $Var^{\mathbb{P}}[G(t)|W(t)] = T^* - t$. An alternative formulation of insider information is one in which insiders know an interval to which a future date's stock price belongs. In this case they enjoy only finite derived utility. See, e.g., Hillairet (2005), Hillairet and Jiao (2017), or D'Auria and Salmerón (2020). Our model shows the possibility that insiders with noisy information can still achieve infinite derived utility, which indicates that even if executives' insider information is noisy, the firm still needs to pursue an effective incentivizing mechanism.

3.2 Insiders' price of risk

Pikovsky and Karatzas (1996) studied several models of insider information, including where insiders observe future returns, or future prices, or noisy future returns. For these models, they showed that the price of risk process for insiders is $\theta_a(t) = \theta(t) + a(t)$, with the $a(t)$ corresponding to the particular model, and is $\theta(t)$ to outsiders.

Corollary 2. The compensating process a, given insider information G in Eq. (5), is

$$a_i(t) = \begin{cases} \frac{\lambda[\lambda(W_1(T^*)-W_1(t))+\sqrt{1-\lambda^2}(W_{d+1}(T^*)-W_{d+1}(t))]}{(T^*-t)} & , \ i = 1 \\ 0 & , \ i = 2, 3, \dots, d \, , \\ \frac{\sqrt{1-\lambda^2}[\lambda(W_1(T^*)-W_1(t))+\sqrt{1-\lambda^2}(W_{d+1}(T^*)-W_{d+1}(t))]}{(T^*-t)} & , \ i = d+1. \end{cases} \tag{6}$$

Proof. Given insider information $G(t)$, write

$$q_t^y \equiv \frac{P(G \in dy|\mathcal{F}_t)}{dy} = \frac{P\{\lambda W_1(T^*) + \sqrt{1-\lambda^2} W_{d+1}(T^*) \in dy \big| W(t)\}}{dy}$$

$$= \frac{1}{\sqrt{2\pi(T^*-t)}} exp\left\{-\frac{\left[y-\left(\lambda W_1(t)+\sqrt{1-\lambda^2}W_{d+1}(t)\right)\right]^2}{2(T^*-t)}\right\}.$$

Using Itô's formula,

$$dq_t^y = \frac{\partial q_t^y}{\partial t} dt + \sum_{i=1}^d \frac{\partial q_t^y}{\partial W_i(t)} dW_i(t) + \frac{1}{2} \sum_{i=1}^d \frac{\partial^2 q_t^y}{\partial [W_i(t)]^2} dt.$$

Taking the derivative of q_t^y w.r.t $W_i(t)$,

$$\frac{\partial q_t^y}{\partial W_1(t)} = q_t^y \times \left(\frac{\lambda\left[y-\left(\lambda W_1(t)+\sqrt{1-\lambda^2}W_{d+1}(t)\right)\right]}{(T^*-t)}\right),$$

$$\frac{\partial q_t^y}{\partial W_i(t)} = 0, i = 2,3, \dots d,$$

$$\frac{\partial q_t^y}{\partial W_{d+1}(t)} = q_t^y \times \left(\frac{\sqrt{1-\lambda^2}\left[y-\left(\lambda W_1(t)+\sqrt{1-\lambda^2}W_{d+1}(t)\right)\right]}{(T^*-t)}\right).$$

Thanks to Jacod (1985) [see also, e.g., Proposition 2.3.2 of Jeanblanc (2010, p. 26)], we have that $\widetilde{W}(t) = W(t) - \int_0^t a(s)ds$ is a $\mathcal{G}$ –martingale, where $a(s)$ is a vector and $\langle q^y, W_i \rangle_t = \int_0^t q_s^y a_i(s)ds$. Substituting $y = \lambda W_1(T^*) + \sqrt{1-\lambda^2} W_{d+1}(T^*)$ gives Eq. (6). □

3.3 Portfolio process

Assume insider executives cannot affect market prices but can dynamically choose a $\mathfrak{R}^{d+1}$ valued $\{\mathcal{G}_t\}$ -progressively measurable portfolio-proportion process $\pi(t) = \left(\pi_1(t), \dots, \pi_{d+1}(t)\right)^\top$, with $\int_0^T \|\pi(t)\|^2 dt < \infty$, almost surely. [See, e.g., Remark 3.6.10, Karatzas and Shreve (1998).]

Namely, they decide at any time $t \in [0, T^*)$ the proportion $\pi_i(t)$ of their wealth $X(t)$ to invest in the ith primary asset, $1 \le i \le d$, based on their enlarged information $\mathcal{G}_t$. The wealth process $X(t)$

corresponding to the portfolio process π follows

$$\begin{aligned} dX(t) &= r(t)X(t)dt + X(t)\pi^\top(t)\sigma(t)\left(\theta_a(t)dt + d\widetilde{W}(t)\right) \\ &= r(t)X(t)dt + X(t)\pi^\top(t)\sigma(t)d\widehat{W}(t), \;\; X(0) = x > 0, \end{aligned} \tag{7}$$

where $\widehat{W}$ is Brownian motion under insider executives' risk neutral probability measure $\mathbb{Q}^{\mathbb{G}}$,

$$\widehat{W}(t) \equiv \widetilde{W}(t) + \int_0^t \theta_a(s)ds. \tag{8}$$

3.4 *Insider portfolio constraints*

As in Cvitanic and Karatzas (1992), for $(t,\omega) \in [0,T] \times \Omega$ we let $K(t,\omega)$ be a closed, convex, nonempty subset of $\Re^{d+1}$. Here K represents the constraint on an executive's portfolio; that is, their portfolio must satisfy $\pi(t) \in K(t,\omega)$ for $(t,\omega) \in [0,T] \times \Omega$. Let

$$\delta(v(t)) \equiv \delta\big(v(t,\omega)\big|K(t,\omega)\big) \triangleq \{\sup_{\varrho\in K}(-\varrho^\top v(t)) \colon \Re^{d+1} \to \Re \cup \{+\infty\}\,; (t,\omega) \in [0,T] \times \Omega\}$$

be the support function of the convex set $-K$, where $v(t) \equiv v(t,\omega) = \big(v_1(t,\omega), \dots, v_{d+1}(t,\omega)\big)^\top$. Let the convex cone

$$\widetilde{K}(t,\omega) \triangleq \{v(t) \in \Re^{d+1};\ \delta(v(t)|K) < \infty\}$$

denote the effective domain of the support function. Let $\mathcal{H}$ denote the Hilbert space of $\{\mathcal{G}_t\}$ – progressively measurable processes v with values in $\Re^{d+1}$ and with the inner product $< v_1, v_2 > \triangleq \mathbb{E}\int_0^T (v_1(t))^\top v_2(t)dt$. Here, K represents the portfolio constraint faced by an executive whose firm's stock is S_1, while $\delta(v(t))$ will be used to solve the constrained portfolio problem.

The constraints placed on an insider executive's portfolio include holding non-transferable non-hedgeable ESOs; a trivial constraint (i.e., no constraint) on other primary assets; and non-tradability on an imaginary primary asset for noise.

Colwell et al. (2015) used a replication argument to translate portfolios with non-transferable non-hedgeable derivatives into portfolios of primary assets (only) with stochastic portfolio constraints. Then the non-transferable non-hedgeable constraint on ESOs can be represented as a constraint on the firm's stock as $[N(t)\Phi(t)S_1(t)/X(t), \infty)$, where $N(t)$ is the number of nonvested ESOs, $\Phi(t)$ is the Black-Scholes delta of the ESO, $S_1(t)$ is the stock price underlying the ESO, and $X(t)$ is the insider executive's total wealth in dollars. Hence, the full portfolio constraint is $K(t,\omega) = [N(t)\Phi(t)S_1(t)/X(t), \infty) \times (-\infty, \infty)^{d-1} \times [0,0]$.

3.5 *Insiders' constrained portfolio optimization*

In this section, we ignore the American feature of the ESOs. We denote the utility function as $U(\cdot)$ and denote the terminal time of portfolio optimization as $T \in [0, T^*]$.

Insiders' derived utility is

$$J^{\mathbb{G}}(x_t, t, T) \equiv \operatorname*{esssup}_{\pi^{\mathbb{G}} \in \mathcal{A}^{\mathbb{G}}(x_t,t,T,K)} \mathbb{E}^{\mathbb{P}}\left[U\left(X^{x_t,\pi^{\mathbb{G}}}(T)\right)\Big|\mathcal{G}_t\right], \tag{9}$$

where $\mathcal{A}^{\mathbb{G}}(x_t,t,T,K)$ is the class of $\Re^{d+1}$ valued $\{\mathcal{G}_t\}$ – progressively measurable portfolio processes,

$\pi^{\mathbb{G}}(t, \omega)$, satisfying the conditions

(i) $\pi^{\mathbb{G}}(t, \omega) \in K$ for $l \otimes \mathbb{P} - a.e.(t, \omega)$,

(ii) $\mathbb{E}^{\mathbb{P}}\left[\max\left(-U\left(X^{x_t, \pi^{\mathbb{G}}}(T)\right), 0\right)\right] < \infty$, and

(iii) for initial capital $x_t \in (0, \infty)$, $X^{x_t, \pi^{\mathbb{G}}}(t) \geq 0$, for $t \in [0, T]$ almost surely.

Similarly, an outsider's derived utility is

$$J^{\mathbb{F}}(x_t, t, T) \equiv \underset{\pi^{\mathbb{F}} \in \mathcal{A}^{\mathbb{F}}(x_t, t, T, K)}{\text{esssup}} \mathbb{E}^{\mathbb{P}}\left[U\left(X^{x_t, \pi^{\mathbb{F}}}(T)\right) \middle| \mathcal{F}_t\right], \tag{10}$$

where $\mathcal{A}^{\mathbb{F}}(x_t, t, T, K)$ is the class of $\Re^{d+1}$ valued $\{\mathcal{F}_t\}$ – progressively measurable portfolio processes, $\pi^{\mathbb{F}}(t, \omega)$, satisfying the conditions

(i) $\pi^{\mathbb{F}}(t, \omega) \in K$ for $l \otimes \mathbb{P} - a.e.(t, \omega)$,

(ii) $\mathbb{E}^{\mathbb{P}}\left[\max\left(-U\left(X^{x_t, \pi^{\mathbb{F}}}(T)\right), 0\right)\right] < \infty$, and

(iii) for initial capital $x_t^{\mathbb{F}} \in (0, \infty)$, $X^{x_t, \pi^{\mathbb{F}}}(t) \geq 0$, for $t \in [0, T]$ almost surely.

Note that $\mathcal{A}^{\mathbb{G}}(x_t, t, T, K)$ and $\mathcal{A}^{\mathbb{F}}(x_t, t, T, K)$ are defined as the sets of admissible portfolio processes from time t to T, to insiders and outsiders, respectively, with the same initial total wealth composition, the same market value x_t, and the same portfolio constraints $K(t, \omega)$. Here, $X^{\mathbb{G}} \equiv X^{x_t, \pi^{\mathbb{G}}}$ is the wealth process given by Eq.(7) corresponding to the portfolio process $\pi^{\mathbb{G}}$ and initial wealth x_t; $X^{\mathbb{F}} \equiv X^{x_t, \pi^{\mathbb{F}}}$ is the solution of same equation corresponding to the portfolio process $\pi^{\mathbb{F}}$ and the same initial wealth; $X^{\mathbb{G}}$ and $X^{\mathbb{F}}$ represent insiders' and outsiders' the total wealth dynamic, respectively.

3.6 Solution of insiders' constrained portfolio optimization

Cvitanić and Karatzas (1992) and Karatzas and Kou (1996) solved the constrained primary assets portfolio optimization problem. By adjusting the drift rates, they transformed the original market into an auxiliary one in which the portfolio constraints automatically hold. The problem becomes a classical unconstrained portfolio optimization, and they show the condition under which the unconstrained solution in the auxiliary market equals the constrained solution in the original market. Colwell et al. (2015) included ESOs in the portfolio by using replication argument to translate ESOs into primary assets and risk-free assets and by analytically pricing non-transferable non-hedgeable American ESOs using the constrained portfolio optimization technique.

Pikovsky and Karatzas (1996) demonstrated the difference between insiders' price of risk, $\theta_a(t) = \theta(t) + a(t)$, and outsiders' price of risk, $\theta(t)$. We further adjust the drift rate owing to the portfolio constraint $K(t, \omega)$ specified in Section 3.4 to transform the insiders' constrained optimization in the original market into a classical unconstrained optimization in the auxiliary market.

From now on, we assume a $ln(\cdot)$ utility. Thanks to Eq. (8.5) and (8.6) in Cvitanić and Karatzas (1992, p. 777), an insider's optimal portfolio process is

$$\pi_v^{\mathbb{G}*}(t) = [\sigma^{\top}(t)]^{-1}\theta_{a,v}(t) = [\sigma(t)\sigma^{\top}(t)]^{-1}\left[b_{a,v}^{\mathbb{G}}(t) - r_v^{\mathbb{G}}(t)\mathbf{1}\right], \tag{11}$$

where $b^{\mathbb{G}}_{a,v}(t) = b(t) + \sigma(t)a(t) + v^{\mathbb{G}}(t) + \delta\left(v^{\mathbb{G}}(t)\right)\mathbf{1}$ and $r^{\mathbb{G}}_{v}(t) = r(t) + \delta\left(v^{\mathbb{G}}(t)\right)$ are the drift rate and the risk-free rate, respectively, in the auxiliary market; $v^{\mathbb{G}}(t)$ is the drift rate adjustment vector owing to the portfolio constraints to transform insiders' perceived market to an auxiliary one; and the scalar support function $\delta(\cdot)$ of the convex set $-K$ is defined in Section 3.4. Then insiders' price of risk considering portfolio constraints is $\theta_{a,v}(t) \equiv \sigma^{-1}(t)\left[b^{\mathbb{G}}_{a,v}(t) - r^{\mathbb{G}}_{v}(t)\mathbf{1}\right]$. Setting $a(t) = 0$ in $\theta_{a,v}$, we obtain outsider executives' price of risk under the same constraints and denote it as

$$\theta_{0,v}(t) = \sigma^{-1}(t)[b(t) + v^{\mathbb{F}}(t) - r(t)\mathbf{1}] = \theta(t) + \sigma^{-1}(t)v^{\mathbb{F}}(t), \tag{12}$$

where $v^{\mathbb{F}}(t)$ is the drift rate adjustment vector owing to the same portfolio constraints to transform outsiders' perceived market to an auxiliary one. Because $a(t) = \left(a_1(t), 0, \dots, 0, a_{d+1}(t)\right)^{\top}$, it follows that

$$\begin{aligned} \sigma(t)a(t) &= \left(\sigma_{11}a_1 + \sigma_{1,d+1}a_{d+1}, \sigma_{21}a_1 + \sigma_{2,d+1}a_{d+1}, \dots, \sigma_{d+1,1}a_1 + \sigma_{d+1,d+1}a_{d+1}\right)^{\top} \\ &= (\sigma_{11}a_1, \sigma_{21}a_1, \dots, \sigma_{d1}a_1, a_{d+1})^{\top}, \end{aligned} \tag{13}$$

where we have used the fact that $\sigma_{i,d+1}(t) = \sigma_{d+1,i}(t) = 0$ for $i \neq d+1$, and $\sigma_{d+1,d+1} = 1$. This implies that insider knowledge about $W_1(T^*)$ can, in general, give executives knowledge about other correlated stocks. It is possible to assume that the insider knowledge is idiosyncratic and, therefore, uncorrelated to other stocks, but we omit the details.

A similar calculation holds for $\sigma^{\top}(t)a(t)$, $\sigma(t)\sigma^{\top}(t)a(t)$ and $\left(\sigma(t)\sigma^{\top}(t)\right)^{-1}a(t)$, as we discuss in the proof of Eq. (15). Also, $\sigma(t)v^{\mathbb{G}}(t) = \left(\sigma_{11}v^{\mathbb{G}}_1, \sigma_{21}v^{\mathbb{G}}_1, \dots, \sigma_{d1}v^{\mathbb{G}}_1, v^{\mathbb{G}}_{d+1}\right)^{\top}$, etc.

If we set $a(t) = 0$, then the insider's constrained portfolio optimization becomes an outsider's one; if we further set $v^{\mathbb{G}}(t) = 0$, then it degenerates into a classical unconstrained portfolio optimization.

We next describe how $v^{\mathbb{G}}(t)$ and the functional form of $\delta(\cdot)$ are determined. Given the constraint $K(t,\omega) = [N(t)\Phi(t)S_1(t)/X(t), \infty) \times (-\infty,\infty)^{d-1} \times [0,0]$, by definition, the corresponding support function is

$$\delta(v(t)) \equiv \sup_{\rho \in K}\left(-\rho^{\top}v\right) = -N(t)\Phi S_1(t)/X(t) \times v_1(t),$$

defined on the effective domain of δ, represented as $\tilde{K} \triangleq \{v(t) \in \Re^{d+1};\ \delta(v(t)|K) < \infty\} = [0,\infty) \times \{0\}^{d-1} \times (-\infty,\infty)$. This implies that $v_2 = \cdots = v_d = 0$ for our problem.

Let the variance-covariance matrix be denoted by $\sigma(t)\sigma^{\top}(t) = \Psi(t)$and let $\Psi^{-1}(t)$, known as the precision matrix, be denoted by $h(t)$, with elements $h_{i,j}(t)$. It is also convenient to write the matrix $\sigma^{-1}(t) =: g(t)$, with elements, $g_{ij}(t)$.

Thanks to Eq. (11.4) in Cvitanić and Karatzas (1992), p. 790, given our value for $\delta(v(t))$ and replacing $\theta(t)$ with $\theta_a(t)$, under ln $(\cdot)$ utility, we have

$$v^{\mathbb{G}}(t) = \underset{v=(v_1,v_2,\dots v_{d+1}) \in \tilde{K}}{\operatorname{argmin}} \left[2\delta(v) + \|\theta_a(t) + \sigma^{-1}(t)v\|^2\right] \tag{14}$$

$$= \underset{v=(v_1,v_2,\ldots v_{d+1})\in\tilde{K}}{\operatorname{argmin}} \left[-\frac{2N(t)\Phi S_1(t)v_1}{X(t)} + \|\sigma^{-1}(t)[b(t)+\sigma(t)a(t)-r(t)\mathbf{1}+v]\|^2 \right].$$

Lemma 1. The solution to Eq. **Error! Reference source not found.**. is

$$v_1^{\mathbb{G}}(t) = \frac{1}{h_{1,1}(t)} \max\left(N(t)\Phi S_1(t)/X(t) - \sum_{i=1}^{d} \big(b_i(t)-r(t)\big)h_{i,1}(t) - g_{11}a_1, 0 \right), \qquad (15)$$

and

$$v_{d+1}^{\mathbb{G}}(t) = r(t) - a_{d+1}(t), \qquad (16)$$

while from the definition of $\tilde{K}$, we get $v_i^{\mathbb{G}}(t) = 0$ for $i = 2, \ldots, d$.

Proof. Recall that in the matrix $\sigma(t)$, $\sigma_{i,d+1}(t) = \sigma_{d+1,i}(t) = 0$ for $i \neq d+1$. This result also holds for $\Psi(t) = \sigma(t)\sigma^{\top}(t)$, as well as for $\Psi^{-1}(t) = h(t)$; that is, $h_{i,d+1}(t) = h_{i,d+1}(t) = 0$, for $i \neq d + 1$. Moreover, $h_{d+1,d+1}(t) = 1$. Next, note that

$$-\frac{2N(t)\Phi S_1(t)v_1}{X(t)} + \|\sigma^{-1}(t)[b(t)+\sigma(t)a(t)-r(t)\mathbf{1}+v]\|^2$$

$$= -\frac{2N(t)\Phi S_1(t)v_1}{X(t)}$$

$$+ (b(t)+\sigma(t)a(t)-r(t)\mathbf{1}+v)^{\top}h(t)(b(t)+\sigma(t)a(t)-r(t)\mathbf{1}+v).$$

Differentiating this with respect to $v_1(t)$ and setting the derivative equal to zero gives

$$-\frac{2N(t)\Phi S_1(t)}{X(t)} + 2(b(t)+\sigma(t)a(t)-r(t)\mathbf{1}+v)^{\top}h(t)e_1$$

$$= -\frac{2N(t)\Phi S_1(t)}{X(t)} + 2(b(t)-r(t)\mathbf{1})^{\top}h(t)e_1 + 2\big(\sigma(t)a(t)\big)^{\top}h(t)e_1 + 2v^{\top}(t)h(t)e_1$$

$$= -\frac{2N(t)\Phi S_1(t)}{X(t)} + 2(b(t)-r(t)\mathbf{1})^{\top}h(t)e_1 + 2(e_1)^{\top}\big(\sigma^{\top}(t)\big)^{-1}a(t) + 2v^{\top}(t)h(t)e_1$$

$$= -\frac{2N(t)\Phi S_1(t)}{X(t)} + 2\sum_{i=1}^{d}(b_i-r)h_{i,1} + 2g_{11}a_1 + 2v_1h_{11} = 0,$$

where $e_j = (0,\ldots,1,\ldots,0)^{\top}$ is a vector with 1 in the j^{th} row and zeroes elsewhere. Here we have used the fact that, for a general matrix m, we have, $(e_i)^{\top}me_j = m_{ij}$. Solving for $v_1(t)$ gives Eq. (15). Finally, because $\tilde{K} = [0,\infty) \times \{0\}^{d-1} \times (-\infty,\infty)$, we see that $v_1(t)$ must remain nonnegative.

Differentiating with respect to v_{d+1} and setting the derivative equal to zero, we find that

$$2([b(t)+\sigma(t)a(t)-r(t)\mathbf{1}])^{\top}h(t)e_{d+1} + 2v(t)h(t)e_{d+1}$$

$$= 2\big(b_{d+1}+\sigma_{d+1,d+1}a_{d+1}-r\big)h_{d+i,d+1} + 2v_{d+1}h_{d+1,d+1} = 2(a_{d+1}-r) + 2v_{d+1} = 0,$$

which prove Eq.(16). □

Setting α = 0 in Eq. (15) and Eq. (16), we get $v^{\mathbb{F}}(t)$.

3.7 Decomposition of utility increment owing to insider information

We discuss how informational advantage can improve insiders' derived utility by decomposing the increment of the derived utility, gained due to insider's information, into two components: a substantial one and a perceived one.

The derived utility increment due to having insider information is $J^{\mathbb{G}}(x_t,t,T) - J^{\mathbb{F}}(x_t,t,T)$.

Under $ln(\cdot)$ utility, it can be decomposed into two components: (i) a *substantial increment,* $\Delta\mathcal{J}(x_t,t,T) \equiv \mathbb{E}^{\mathbb{P}}(\Delta J(x_t,t,T)|\mathcal{F}_t)$, owing to insiders' ability to improve the optimal portfolio process conditional on an enlarged information set, where $\Delta J(x_t,t,T) \equiv J^{\mathbb{G}}(x_t,t,T) - J^{\mathbb{F}}(x_t,t,T)$, and (ii) a *perceived component,* $\Delta J(x_t,t,T) - \Delta\mathcal{J}(x_t,t,T)$, caused by insiders' and outsiders' differing perceptions. If the American features of ESOs are further considered (see Section 4.2), the perceived component affects insiders' decisions on choosing the optimal exercise time (or optimal exercise rate if partial exercise is allowed) of their ESO. Because ESOs are non-transferable non-hedgeable, exercising ESOs relaxes the portfolio constraints, which alters the insider's optimal constrained portfolio process and, accordingly, causes a substantial impact.

As an illustration, solving Eq. (7), we get

$$X^{x_t,\pi}(T) = x_t \exp\left[\int_t^T \left(r(u) - \tfrac{1}{2}\|\sigma^\top(u)\pi(u)\|^2 + \pi^\top(u)\sigma(u)\theta_a(u)\right)du + \int_t^T \pi^\top(u)\sigma(u)d\widetilde{W}(u)\right],$$

$$= x_t \exp\left[\int_t^T \left(r(u) + \frac{1}{2}\|\theta_a(u)\|^2 - \frac{1}{2}\|\theta_a(u) - \sigma^\top(u)\pi(u)\|^2\right)du \right. \qquad (17)$$

$$\left. + \int_t^T \pi^\top(u)\sigma(u)d\widetilde{W}(u)\right].$$

For log utility, the insiders' optimal constrained portfolio process is given by $\pi^{\mathbb{G}}(u) = [\sigma^\top(u)]^{-1}\theta_{a,v}(u)$. Recalling that $\theta_{a,v}(u) = \theta(u) + a(u) + \sigma^{-1}(u)v^{\mathbb{G}}(u) = \theta_a(u) + \sigma^{-1}(u)v^{\mathbb{G}}(u)$, as defined in Corollary 1, we see that $\theta_a(u) - \sigma^\top(u)\pi^{\mathbb{G}}(u) = \theta_a(u) - \theta_{a,v}(u) = -\sigma^{-1}(u)v^{\mathbb{G}}(u)$. Substituting into Eq.(17) and taking logs gives

$$J^{\mathbb{G}}(x_t,t,T) \equiv ln(x_t) + \mathbb{E}^{\mathbb{P}}\left[\int_t^T \left(r(u) + \tfrac{1}{2}\|\theta_a(u)\|^2\right)du \,\middle|\, \mathcal{G}_t\right] \qquad (18)$$

$$-\mathbb{E}^{\mathbb{P}}\left[\int_t^T \left(\tfrac{1}{2}\left\|\sigma^{-1}(u)v^{\mathbb{G}}(u)\right\|^2\right)du \,\middle|\, \mathcal{G}_t\right]$$

$$= ln(x_t) + \mathbb{E}^{\mathbb{P}}\left[\int_t^T \left(r(u) + \tfrac{1}{2}\|\theta(u)\|^2\right)du \,\middle|\, \mathcal{G}_t\right] + \Delta J^{asy} + \Delta J^{sym}$$

$$-\mathbb{E}^{\mathbb{P}}\left[\int_t^T \left(\tfrac{1}{2}\left\|\sigma^{-1}(u)v^{\mathbb{G}}(u)\right\|^2\right)du \,\middle|\, \mathcal{G}_t\right],$$

where $\Delta J^{asy} \equiv \mathbb{E}^{\mathbb{P}}\left[\int_t^T \theta^\top(u)a(u)du \,\middle|\, \mathcal{G}_t\right]$ and $\Delta J^{sym} \equiv \mathbb{E}^{\mathbb{P}}\left[\int_t^T \tfrac{1}{2}\|a(u)\|^2 du \,\middle|\, \mathcal{G}_t\right]$.

Setting $a(u) = 0$ and $v^{\mathbb{G}}(u) = 0$ for $u \in [t,T]$ and changing $\mathcal{G}_t$ into $\mathcal{F}_t$ in Eq.(18), we get an outsider investor's derived utility,

$$J^{\mathbb{F}}(x_t,t,T) \equiv ln(x_t) + \mathbb{E}^{\mathbb{P}}\left[\int_t^T \left(r(u) + \tfrac{1}{2}\|\theta(u)\|^2\right)du \,\middle|\, \mathcal{F}_t\right]. \qquad (19)$$

To highlight the effect of insider information, let us ignore insider executives' portfolio constraints; that is, set $v^{\mathbb{G}}(u) = 0$ in Eq. (18). Subtracting Eq. (19) from Eq.(18), we see that the substantial increment owing to insider information can be decomposed into a *symmetric impact* $\mathbb{E}^{\mathbb{P}}(\Delta J^{sym}(t)|\mathcal{F}_t)$and an *asymmetric impact* $\mathbb{E}^{\mathbb{P}}(\Delta J^{asy}(t)|\mathcal{F}_t)$. The symmetric impact, determined by the squared norm of the compensating process, implies the same benefit to the insiders whether the (noisy) information indicates good news, i.e., $W_1(T^*) > W_1(t)$, or bad news, i.e., $W_1(T^*) < W_1(t)$. The asymmetric impact becomes zero, if, for any time, $u \in [t,T]$, $a(u)$ is orthogonal to $\mathcal{F}_u$; otherwise, good news and bad news can cause an unequal impact on insiders' derived utility.

Whether $a(u)$ is orthogonal to $\mathcal{F}_u$, is determined by the type of insider information. For example, if insiders know the (noisy) terminal stock price, then the substantial increment has only a symmetric impact; however, if they know the peak of the stock's return, within a term up to T^*, then good news or bad news causes an asymmetric impact on insiders' derived utilities.

In our case, with $a(u)$ given by Eq. (6) in Corollary 2, $a(u)$ is indeed orthogonal to $\mathcal{F}_u$, so the asymmetric impact is equal to zero when $v^{\mathbb{G}}(u) = 0$. Intuitively, if there are no portfolio constraints, an insider can make money even when the news is bad, say, by shorting an asset. On the other hand, for an insider with portfolio constraints given by $K(t, \omega)$, $v^{\mathbb{G}}(t)$ is actually a function of $a(t)$, so the asymmetric impact is not equal to zero, in general, even when $a(u)$ is orthogonal to $\mathcal{F}_u$.

4 Policy recommendations on blackout trading period regulation

We discuss four practical questions. (1) Is a blackout still required when insider information is (very) noisy and insider executives must obey non-transferable non-hedgeable constraints? (2) How long should a blackout be? (3) Who should take the role of choice entity to enact a blackout, and who should mandate it? (4) What assets should blackout trade prohibitions include?

4.1 The necessity of blackout trading periods

Under the model in Section 3, we have the following proposition.

Proposition 1. Under $ln\,(\cdot)$ utility, let

$$G(t) \equiv W_1(t) + \lambda[W_1(T^*) - W_1(t)] + \sqrt{1-\lambda^2}[W_{d+1}(T^*) - W_{d+1}(t)]$$

for all $t \in [0, T]$, with portfolio constraints

$$K(t, \omega) = [(N(t))\Phi S_1(t)/X(t), \infty) \times (-\infty, \infty)^{d-1} \times [0,0].$$

Then,

$$\Delta\mathcal{J}(x_t, t, T) \geq \frac{1}{2} ln\left(\frac{(T^*-t)}{(T^*-T)}\right) - \frac{1}{2}C_1(t),$$

where $C_1(t) = \mathbb{E}^{\mathbb{P}}\left[\int_t^T \left(C_0(u)\right)^2 h_{1,1}(u)du \middle| \mathcal{F}_t\right] + \mathbb{E}^{\mathbb{P}}\left[\int_t^T \left(r(u)\right)^2 du \middle| \mathcal{F}_t\right]$, with

$$C_0(u) = \frac{1}{h_{1,1}(u)}\left(N(u)\Phi S_1(u)/\mathbb{X}(u) - \sum_{i=1}^{d}(b_i(u) - r(u))h_{i,1}(u)\right).$$

If $C_1(t) < \infty$, then $\Delta\mathcal{J}(x_t, t, T) \to \infty$, as $T \to T^*$.

Proof. If we write $\alpha(u) = \frac{[\lambda(W_1(T^*)-W_1(u))+\sqrt{1-\lambda^2}(W_{d+1}(T^*)-W_{d+1}(u))]}{(T^*-u)}$, then, $a_1(u) = \lambda\alpha(u)$ and $a_{d+1}(u) = \sqrt{1-\lambda^2}\alpha(u)$. Note that, for $t \leq u$, $\mathbb{E}^{\mathbb{P}}[\alpha(u)|\mathcal{F}_t] = 0$ and $\mathbb{E}^{\mathbb{P}}\left[\left(\alpha(u)\right)^2 \middle| \mathcal{F}_t\right] = \frac{(T^*-u)}{(T^*-u)^2} = \frac{1}{(T^*-u)}$, which implies that $\mathbb{E}^{\mathbb{P}}\left[\int_t^T \left(\alpha(u)\right)^2 \middle| \mathcal{F}_t\right] = \int_t^T \frac{1}{(T^*-u)}du = ln\left(\frac{(T^*-t)}{(T^*-T)}\right)$. In fact,

$$\mathbb{E}^{\mathbb{P}}\left[\int_t^T \frac{1}{2}\|a(u)\|^2 du \middle| \mathcal{F}_t\right] = \frac{1}{2}\mathbb{E}^{\mathbb{P}}\left[\int_t^T [(a_1(u))^2 + (a_{d+1}(u))^2]du \middle| \mathcal{F}_t\right] = \frac{1}{2} ln\left(\frac{(T^*-t)}{(T^*-T)}\right).$$

Moreover, because $a(u)$ is orthogonal to $\mathcal{F}_u$, for any sufficiently integrable, $\{\mathcal{F}_t\}$-adapted process, $\hat{b}$ $\mathbb{E}^{\mathbb{P}}[\alpha(u)\hat{b}(u)|\mathcal{F}_t] = \mathbb{E}^{\mathbb{P}}[\mathbb{E}^{\mathbb{P}}[\alpha(u)\hat{b}(u)|\mathcal{F}_u]|\mathcal{F}_t] = \mathbb{E}^{\mathbb{P}}[\hat{b}(u)\mathbb{E}^{\mathbb{P}}[\alpha(u)|\mathcal{F}_u]|\mathcal{F}_t] = 0.$

From Eq.(18) and Eq. (19),

$$\mathbb{E}^{\mathbb{P}}[\Delta J(x_t,t,T)|\mathcal{F}_t] = \mathbb{E}^{\mathbb{P}}\left[J^{\mathbb{G}}(x_t,t,T) - J^{\mathbb{F}}(x_t,t,T)\middle|\mathcal{F}_t\right]$$

$$= \mathbb{E}^{\mathbb{P}}(\Delta J^{asy}(t)|\mathcal{F}_t) + \mathbb{E}^{\mathbb{P}}(\Delta J^{sym}(t)|\mathcal{F}_t) - \mathbb{E}^{\mathbb{P}}\left[\int_t^T\left(\frac{1}{2}\left\|\sigma^{-1}(u)v^{\mathbb{G}}(u)\right\|^2\right)du\middle|\mathcal{F}_t\right]$$

$$= \mathbb{E}^{\mathbb{P}}\left[\int_t^T \theta^{\top}(u)a(u)du\middle|\mathcal{F}_t\right] + \mathbb{E}^{\mathbb{P}}\left[\int_t^T \frac{1}{2}\|a(u)\|^2du\middle|\mathcal{F}_t\right]$$

$$-\mathbb{E}^{\mathbb{P}}\left[\int_t^T\left(\frac{1}{2}\left\|\sigma^{-1}(u)v^{\mathbb{G}}(u)\right\|^2\right)du\middle|\mathcal{F}_t\right]$$

$$= 0 + \frac{1}{2}ln\left(\frac{(T^*-t)}{(T^*-T)}\right) - \mathbb{E}^{\mathbb{P}}\left[\int_t^T\left(\frac{1}{2}\left\|\sigma^{-1}(u)v^{\mathbb{G}}(u)\right\|^2\right)du\middle|\mathcal{F}_t\right].$$

Next, recall that we are writing the i,j^{th} element of $\sigma^{-1}(u)$ as g_{ij}. Because $\sigma_{i,d+1} = \sigma_{d+1,i} = 0$ for $i \neq d+1$, it follows that $g_{i,d+1} = g_{d+1,i} = 0$ for $i \neq d+1$, as well. Thus,

$$\sigma^{-1}(u)v^{\mathbb{G}}(u) = \left(g_{11}v_1^{\mathbb{G}} + g_{1,d+1}v_{d+1}^{\mathbb{G}}, \dots, g_{d,1}v_1^{\mathbb{G}} + g_{d,d+1}v_{d+1}^{\mathbb{G}}, g_{1,d+1}v_1^{\mathbb{G}} + g_{d+1,d+1}v_{d+1}^{\mathbb{G}}\right)^{\top}$$

$$= \left(g_{11}v_1^{\mathbb{G}}, \dots, g_{d,1}v_1^{\mathbb{G}}, g_{d+1,d+1}v_{d+1}^{\mathbb{G}}\right)^{\top} = \left(g_{11}v_1^{\mathbb{G}}, \dots, g_{d,1}v_1^{\mathbb{G}}, v_{d+1}^{\mathbb{G}}\right)^{\top},$$

as $g_{d+1,d+1} = 1$, by assumption. Note that $h = g^{\top}g$, and so, $h_{1,1} = \sum_{i=1}^{d+1}\left(g_{i,1}\right)^2 = \sum_{i=1}^{d}\left(g_{i,1}\right)^2$, which implies that $\frac{\left(g_{1,1}(u)\right)^2}{h_{1,1}(u)} \leq 1$.

Now,

$$\left\|\sigma^{-1}(u)v^{\mathbb{G}}(u)\right\|^2 = \left(v_1^{\mathbb{G}}\right)^2\sum_{i=1}^{d}\left(g_{i,1}\right)^2 + \left(v_{d+1}^{\mathbb{G}}\right)^2 = \left(v_1^{\mathbb{G}}\right)^2 h_{1,1} + \left(v_{d+1}^{\mathbb{G}}\right)^2.$$

Note that

$$v_1^{\mathbb{G}}(u) = \left(C_0(u) - \frac{g_{1,1}(u)}{h_{1,1}(u)}a_1(u)\right)^+.$$

Then,

$$\mathbb{E}^{\mathbb{P}}\left[\left(v_1^{\mathbb{G}}(u)\right)^2 h_{1,1}(u)\middle|\mathcal{F}_t\right] = \mathbb{E}^{\mathbb{P}}\left[\left(C_0(u) - \frac{g_{1,1}(u)}{h_{1,1}(u)}a_1(u)\right)^2 1_{\{v_1^{\mathbb{G}}(u)>0\}}h_{1,1}(u)\middle|\mathcal{F}_t\right]$$

$$\leq \mathbb{E}^{\mathbb{P}}\left[\left(C_0(u) - \frac{g_{1,1}(u)}{h_{1,1}(u)}a_1(u)\right)^2 h_{1,1}(u)\middle|\mathcal{F}_t\right]$$

$$= \mathbb{E}^{\mathbb{P}}\left[\left(C_0(u)\right)^2 h_{1,1}(u) + \left(\frac{g_{1,1}(u)}{h_{1,1}(u)}a_1(u)\right)^2 h_{1,1}(u)\middle|\mathcal{F}_t\right]$$

$$= \mathbb{E}^{\mathbb{P}}\left[\left(C_0(u)\right)^2 h_{1,1}(u)\middle|\mathcal{F}_t\right] + \frac{\lambda}{(T^*-u)}\mathbb{E}^{\mathbb{P}}\left[\frac{\left(g_{1,1}(u)\right)^2}{h_{1,1}(u)}\middle|\mathcal{F}_t\right],$$

and

$$\mathbb{E}^{\mathbb{P}}\left[\left(v_{d+1}^{\mathbb{G}}(u)\right)^2\middle|\mathcal{F}_t\right] = \mathbb{E}^{\mathbb{P}}\left[\left(r(u) - a_{d+1}(u)\right)^2\middle|\mathcal{F}_t\right]$$

$$= \mathbb{E}^{\mathbb{P}}[(r(u))^2|\mathcal{F}_t] - \mathbb{E}^{\mathbb{P}}\left[\left(a_{d+1}(u)\right)^2\middle|\mathcal{F}_t\right]$$

$$= \mathbb{E}^{\mathbb{P}}[(r(u))^2|\mathcal{F}_t] - \frac{(1-\lambda^2)}{(T^*-u)}.$$

So,

$$\mathbb{E}^{\mathbb{P}}\left[\int_t^T\left(\frac{1}{2}\left\|\sigma^{-1}(u)v^{\mathbb{G}}(u)\right\|^2\right)du\middle|\mathcal{F}_t\right]\leq\mathbb{E}^{\mathbb{P}}\left[\frac{1}{2}\int_t^T\left(C_0(u)\right)^2h_{1,1}(u)du\middle|\mathcal{F}_t\right]$$

$$+\mathbb{E}^{\mathbb{P}}\left[\frac{1}{2}\int_t^T\frac{\left(g_{1,1}(u)\right)^2}{h_{1,1}(u)}\frac{(1-\lambda^2)}{(T^*-u)}du\middle|\mathcal{F}_t\right]$$

$$+\frac{1}{2}\ \mathbb{E}^{\mathbb{P}}\left[\int_t^T(r(u))^2du\middle|\mathcal{F}_t\right]-(1-\lambda^2)\frac{1}{2}ln\left(\frac{(T^*-t)}{(T^*-T)}\right).$$

Therefore, given that $\frac{\left(g_{1,1}(u)\right)^2}{h_{1,1}(u)}\leq 1$, we have

$$\mathbb{E}^{\mathbb{P}}[\Delta J(x_t,t,T)|\mathcal{F}_t]\ \geq\frac{1}{2}ln\left(\frac{(T^*-t)}{(T^*-T)}\right)-\mathbb{E}^{\mathbb{P}}\left[\frac{1}{2}\int_t^T\left(C_0(u)\right)^2h_{1,1}(u)du\middle|\mathcal{F}_t\right]$$

$$-\mathbb{E}^{\mathbb{P}}\left[\frac{1}{2}\int_t^T\frac{\left(g_{1,1}(u)\right)^2}{h_{1,1}(u)}\frac{(1-\lambda^2)}{(T^*-u)}du\middle|\mathcal{F}_t\right]$$

$$-\frac{1}{2}\ \mathbb{E}^{\mathbb{P}}\left[\int_t^T\left(r(u)\right)^2du\middle|\mathcal{F}_t\right]+(1-\lambda^2)\frac{1}{2}ln\left(\frac{(T^*-t)}{(T^*-T)}\right)$$

$$\geq\frac{1}{2}ln\left(\frac{(T^*-t)}{(T^*-T)}\right)-\frac{1}{2}C_1(t),$$

as desired. □

Proposition 1 sheds light on three issues. First, it supports the legal basis for implementing blackout trading periods, without which noisy insider information at any level of quality, except pure noise, invalidates firms' incentives for executives the same way as accurate insider information does, even with a binding non-transferable non-hedgeable portfolio constraint. Second, insiders' substantial increment of derived utility (comparing to outsiders') increases with respect to a portfolio holding period T at a speed proportional to information quality, which indicates that the better the information quality insiders hold, the longer the blackout must be to effectively prevent insiders from obtaining derived utility above a certain level. Finally, if the insider information is correlated with at least one additional firm, insiders can achieve infinite derived utility.

Given executive attributes, we should identify blackout trading period regulatory schemes that prevent the harmful effects of insider trading (particularly, nullifying aligning incentives).

4.2 Inadequacy and excessiveness of blackout trading period

We define the *lower bound* ($\mathfrak{B}^L$) of a blackout as a period such that any shorter blackout is considered inadequate, in the sense that it would induce executive insiders to instantaneously exercise all their ESOs once vested, effectively nullifying the incentivizing mechanism.

We define the *upper bound* ($\mathfrak{B}^U$) of a blackout as a period such that any longer blackout is considered as excessive, in the sense that it makes insiders worse off than outsiders due to the tightened constraints, despite their informational advantage, which is unfair and harms markets' informational efficiency. An *effective blackout* is defined as any period, $\mathfrak{B}$, satisfying $\mathfrak{B}\in(\mathfrak{B}^L,\mathfrak{B}^U]$.

We consider an insider executive's portfolio that includes ESOs with a continuous partial exercise feature. Assume n is the number of ESOs initially granted to executives and t_{vo} is the end of the option vesting period. The optimal exercise rate process $\dot{n}(t)$ determines the number of remaining

ESOs still under portfolio constraints; the number of options exercised by date t is $\widehat{N}(t) \coloneqq \int_0^t \dot{n}(s)ds$, and we write $N(t) = n - \widehat{N}(t)$ for the number of options still held at date t. Then, the exercise rate $\dot{n}(t)$ belongs to the set $\mathcal{N}(t)$, the collection of all feasible $\{\mathcal{F}_t\}$ – progressively measurable exercise rates, where $\mathcal{N}(t) \triangleq \left\{\dot{n}(t): \dot{n}(t) = 0 \text{ for } t < t_{vo}, \dot{n}(t) \geq 0 \text{ for } t \geq t_{vo}, \text{and } \int_0^t \dot{n}(s)ds \leq n \text{ for } t \geq 0\right\}$. The portfolio constraint at time t is

$$K(t,\omega) = \left[\left(n - \widehat{N}(t)\right)\Phi(t)S_1(t)/X(t), \infty\right) \times (-\infty,\infty)^{d-1} \times [0,0]. \tag{20}$$

The optimal portfolio process $\pi^{\mathbb{G}*}$ and exercise rate process $\dot{n}^*$ jointly solve

$$\mathbb{J}^{\mathbb{G}} \equiv \mathbb{J}^{\mathbb{G}}\left(X^{*,\dot{n}^*}(t), \widehat{N}(t), S(t), \lambda, t, T\right) \equiv \underset{(\pi,\dot{n})\in\left(\mathcal{A}^{\mathbb{G}}(x_t,t,T,K)\times\mathcal{N}(t)\right)}{\operatorname{esssup}} \mathbb{E}^{\mathbb{P}}\left[U\left(X^{x_t,\pi^{\mathbb{G}}}(T)\right)\middle|\,\mathcal{G}_t\right]. \tag{21}$$

The following proposition provides conditions under which there is a blackout trading period lower bound.

Proposition 2. Suppose $\frac{1}{h_{1,1}(t)}\sum_{i=1}^d \left(b_i(t) - r(t)\right)h_{i,1}(t) + g_{11}a_1 < 0$ for all $0 \leq t \leq T$. Then there exists a $\mathfrak{B}^L$ such that for any $T \in [T^* - \mathfrak{B}^L, T^*]$, once the ESOs are vested at t_{vo}, $\dot{n}^*(t_{vo})$, approaches infinity; that is, $N(t) = 0$ is optimal for all $0 \leq t \leq T$. This case is more likely if the insider information is bad news, i.e., if $a_1 < 0$. On the other hand, if $\frac{1}{h_{1,1}(t)}\sum_{i=1}^d \left(b_i(t) - r(t)\right)h_{i,1}(t) + g_{11}a_1 > 0$ for all $0 \leq t \leq T$, then $N(t) = 0$ is never optimal. This case is more likely if the insider information is good news, i.e., if $a_1 > 0$.

Proof. We denote by $X^{*,\dot{n}}$ the total wealth process generated by the optimal portfolio process $\pi^{\mathbb{G}*}$ for a given initial wealth x_t and under an exercise rate process $\dot{n}^*$. Eq. (35) in Colwell et al. (2015, p. 168) gives the first-order condition solving the stochastic control problem to justify $\dot{n}^*(t)$. Under insiders' enlarged filtration, drift rates are altered; however, the following FOC still applies:

$$-\left(BS(t) - B(t)\right)\frac{\partial \mathbb{J}^{\mathbb{G}}}{\partial X^{*,\dot{n}}(t)} + \frac{\partial \mathbb{J}^{\mathbb{G}}}{\partial \widehat{N}(t)} = 0, \tag{22}$$

where $BS(t)$ represents the theoretical price of the European call option, and it is the Black-Scholes price when $r(t)$, $\sigma(t)$ are deterministic; $B(t)$ is the option's intrinsic value. It is well known that without any portfolio constraints, an American call option on a non-dividend-paying stock should not be exercised early. The financial intuition of Eq. (22) is that the optimal exercise rate is set so that the marginal utility increase due to partial elimination of the constraint exactly offsets the marginal utility decrease from the time-value loss from early exercise.

For this proof, it is convenient to write $Y(u) = \frac{N(u)\Phi S_1(u)}{h_{1,1}(u)X^{*,\dot{n}}(u)}$. Recall that, $N(u) = n - \widehat{N}(u) = n - \widehat{N}(t) - \int_t^u \dot{n}(s)ds$. Next, from Eq. (15), let us write

$$v_1^{\mathbb{G}}(t) = \max(Y(t) - A(t), 0) = (Y(t) - A(t))^+,$$

where

$$A(t) = \frac{1}{h_{1,1}(t)}\left(\sum_{i=1}^{d}\big(b_i(t) - r(t)\big)h_{i,1}(t) + g_{11}a_1\right).$$

Then,

$$\mathbb{E}^{\mathbb{P}}\left[\left(v_1^{\mathbb{G}}(u)\right)^2 h_{1,1}(u)\middle|\mathcal{G}_t\right] = \mathbb{E}^{\mathbb{P}}\left[(Y(u) - A(u))^2 1_{\{Y(u)>A(u)\}}h_{1,1}(u)\middle|\mathcal{G}_t\right].$$

In general, for a random variable, U, with probability density function $f_U(x)$,

$\frac{\partial}{\partial A}\mathbb{E}^{\mathbb{P}}\left[g(A,U)1_{\{U>A\}}\right] = \frac{\partial}{\partial A}\int_A^\infty g(A,x)f_U(x)dx = \int_A^\infty \frac{\partial}{\partial A}g(A,x)f_U(x)dx - g(A,A)f_U(A).$

In our case, $g(A,A) = (A(u) - A(u))^2 = 0$, and so,

$$\begin{aligned}
&\frac{\partial}{\partial X^{*,\dot{n}}(t)}\mathbb{E}^{\mathbb{P}}\left[(Y(u) - A(u))^2 1_{\{Y(u)>A(u)\}}h_{1,1}(u)\middle|\mathcal{G}_t\right] \\
&\quad = 2\mathbb{E}^{\mathbb{P}}\left[\big(Y(u) - A(u)\big)\frac{\partial}{\partial X^{*,\dot{n}}(t)}Y(u)1_{\{Y(u)>A(u)\}}h_{1,1}(u)\middle|\mathcal{G}_t\right] \\
&\quad = -2\mathbb{E}^{\mathbb{P}}\left[\big(Y(u) - A(u)\big)\frac{N(u)\Phi S_1(u)}{h_{1,1}(u)\left(X^{*,\dot{n}}(u)\right)^2}\frac{\partial X^{*,\dot{n}}(u)}{\partial X^{*,\dot{n}}(t)}1_{\{Y(u)>A(u)\}}h_{1,1}(u)\middle|\mathcal{G}_t\right],
\end{aligned}$$

which implies that

$$\frac{\partial \mathbb{J}^{\mathbb{G}}}{\partial X^{*,\dot{n}}(t)} = \frac{1}{X^{*,\dot{n}}(t)} + \mathbb{E}^{\mathbb{P}}\left[\big(Y(u) - A(u)\big)\frac{N(u)\Phi S_1(u)}{h_{1,1}(u)\left(X^{*,\dot{n}}(u)\right)^2}\frac{\partial X^{*,\dot{n}}(u)}{\partial X^{*,\dot{n}}(t)}1_{\{Y(u)>A(u)\}}h_{1,1}(u)\middle|\mathcal{G}_t\right].$$

All else being equal, as $N(u)$ increases, $\frac{\partial \mathbb{J}^{\mathbb{G}}}{\partial X^{*,\dot{n}}(t)}$ increases.

Similarly,

$$\begin{aligned}
\frac{\partial \mathbb{J}^{\mathbb{G}}}{\partial \hat{N}(t)} &= -\frac{1}{2}\frac{\partial}{\partial \hat{N}(t)}\mathbb{E}^{\mathbb{P}}\left[\int_t^T (Y(u) - A(u))^2\, 1_{\{Y(u)>A(u)\}}h_{1,1}(u)du\middle|\mathcal{G}_t\right] \\
&= \mathbb{E}^{\mathbb{P}}\left[\int_t^T \big(Y(u) - A(u)\big)\frac{\Phi S_1(u)}{X^{*,\dot{n}}(u)}1_{\{Y(u)>A(u)\}}du\middle|\mathcal{G}_t\right].
\end{aligned}$$

From Eq. (22),

$$\big(BS(t) - B(t)\big)\frac{\partial \mathbb{J}^{\mathbb{G}}}{\partial X^{*,\dot{n}}(t)} = \frac{\partial \mathbb{J}^{\mathbb{G}}}{\partial \hat{N}(t)},$$

and if $N(t_{vo}) = 0$, it follows that

$$\big(BS(t_{vo}) - B(t_{vo})\big)\frac{1}{X^{*,\dot{n}}(t_{vo})} = \mathbb{E}^{\mathbb{P}}\left[\int_{t_{vo}}^T \big(-A(u)\big)\frac{\Phi S_1(u)}{X^{*,\dot{n}}(u)}1_{\{0>A(u)\}}du\middle|\mathcal{G}_{t_{vo}}\right].$$

Keeping the option on the left-hand side fixed, the integrand on the right-hand side is nonnegative and equal to zero at $T = t_{vo}$. If the insider information is good news, i.e., if $a_1 > 0$, then, depending on the other parameters, it is possible that $A(u) > 0$ for all u, in which case the right hand side of the above equation would be equal to zero and $N(t) = 0$ would never be a solution to the first order conditions. If, on the other hand, $A(u) < 0$ for all u, then there might exist a time, denoted by $\hat{T}$, say, such that the first-order condition holds for $N(\hat{T}) = 0$. In this case, we set $\mathfrak{B}^L = T^* - \hat{T}$. □

Proposition 2 demonstrates that if an insider's information is good news, then they are more likely to hold onto their ESOs rather than exercising them, and if their information is bad news, they are more likely to exercise the ESOs sooner, effectively freeing up their portfolio constraints.

We define the upper bound of blackouts as the one making insider executives' substantial increment of the derived utility equal to zero, when outsiders can freely trade until T^* and the terminal trading time of insiders is just before the blackout starts, assuming they have same cash-only endowments.

Proposition 3. There exists a unique $\mathfrak{B}^U$ such that, at $T \equiv T^* - \mathfrak{B}^U$,

$$\mathbb{E}^{\mathbb{P}}\left[J^{\mathbb{G}}(x_t, t, T) - J^{\mathbb{F}}(x_t, t, T^*)\middle|\mathcal{F}_t\right]=0.$$

Proof. Given G under our setting, $\mathbb{E}^{\mathbb{P}}\left[\mathbb{J}^{\mathbb{G}}(x_t, t\ T)\middle|\mathcal{F}_t\right] \to \infty$ as $T \to T^*$. Also, for $T = t$,

$$\mathbb{E}^{\mathbb{P}}\left[J^{\mathbb{G}}(x_t, t, t) - J^{\mathbb{F}}(x_t, t, t)\middle|\mathcal{F}_t\right] = 0;$$

and because $J^{\mathbb{F}}(x_t, t, T) < J^{\mathbb{F}}(x_t, t, T^*)$ for any $T < T^*$, it follows that

$$\mathbb{E}^{\mathbb{P}}\left[J^{\mathbb{G}}(x_t, t, t) - J^{\mathbb{F}}(x_t, t, T^*)\middle|\mathcal{F}_t\right] < 0.$$

Finally, $\mathbb{E}^{\mathbb{P}}\left[J^{\mathbb{G}}(x_t, t, T)\middle|\mathcal{F}_t\right]$ is continuous and monotone increasing in *T*, which proves the result. □

4.3 Optimal regulations

The analysis in the previous section demonstrates that the optimal boundaries of blackouts are conditional on executives' specific attributes, including the nature of their information (the type, quality, and dates of future events that give them improved predictive power), total wealth, wealth composition, and portfolio constraints. As firms have incentives to establish effective blackouts, it becomes apparent that firms' superior relevant information on these attributes makes them the best entity, rather than the SEC or other regulators, to mandate blackout-trading-period boundaries.

4.4 Blackout trading prohibited asset list

A relevant question is, what assets should blackout trading prohibitions include? Should they include firms' securities (i.e., stock and ESOs) only or additional components of executives' other wealth? The following proposition demonstrates that the asset list depends on whether the risk that comprises the insider information is idiosyncratic or systematic.

We consider a scenario in which, within the firm, there are some executives who have insider information and others who do not, and they hold the same portfolio with weight $\phi \equiv (\phi_1, \dots, \phi_{d+1})$ when the blackout starts. Because it is hard in practice to screen insiders from outsiders and almost impossible to implement a blackout trading prohibition rule on insiders and outsiders differently, we seek an asset list that applies equally to executives during a blackout.

In the following proposition, we assume that the individual who does not have insider information still has the same portfolio constraint as the insider who does.

Proposition 4. Given $T \in [t_{\mathfrak{B}}, T^*]$, where $t_{\mathfrak{B}} \equiv T^* - \mathfrak{B}$, then, $\forall t \in [t_{\mathfrak{B}}, T]$ and $\forall u \in [t, T]$, for any type of G and corresponding $a(t)$,

i. if $\forall\, i = 2, \dots, d,\ \sigma_{i,1}(u) = 0$, and $K(u, \omega) = [\phi_1(u), \phi_1(u)] \times (-\infty, \infty)^{d-1}$, then $\Delta\mathcal{J}(x_t, t, T) = 0$;

ii. if $\exists\, i = 2, \ldots, d, s.t. \sigma_{i,1}(u) \neq 0$ and $K(u, \omega) = [\phi_1(u), \phi_1(u)] \times (-\infty, \infty)^{d-1}$, then $\Delta\mathcal{J}(x_t, t, T) \to \infty$, as $T \to T^*$;

iii. if $\exists\, i = 2, \ldots, d, s.t. \sigma_{i,1}(u) \neq 0$ and $K(u, \omega) = [\phi_1(u), \phi_1(u)] \times \ldots \times [\phi_d(u), \phi_d(u)]$, then $\Delta\mathcal{J}(x_t, t, T) = 0$.

Proof. See Appendix.

Although insider and outsider executives are facing the same portfolio constraints, the strength of the constraints could be different. For example, if insiders know the firm's stock will rise or decline for sure and they cannot vary their position during a blackout, then the insiders' constraint is effectively stricter than that of outsiders. Two facts reflect this point: $v^{\mathbb{G}}(\cdot)$ generally is a function of $a(\cdot)$, and there is a value difference between $v^{\mathbb{G}}(\cdot)$ and $v^{\mathbb{F}}(\cdot)$.

The financial intuition of Proposition 4 is that to prevent insiders from getting extra substantial utility after a blackout trading period starts,

i. if the insider information is purely idiosyncratic, then the firm should list only the firm's stock on the blackout trading prohibition list. The disadvantage from having stricter portfolio constraints caused by blackout trading periods offsets the advantage of possessing insider information. Consequently, there is no *substantial increment* of insiders' derived utility compared to that of outsider executives.

ii. if the insider information is not purely idiosyncratic,[11] insider executives can acquire infinite derived utility by trading other firms' shares, even when restricted from trading their own firms' shares. Thus, the SEC or other regulators should restrict trading in all firms' shares during blackouts.

Please note that although our model assumes log utility and a specific insider information type, Proposition 4 is derived without needing those specifications. Hence, they are safe to use as a legal basis generally for enacting blackout trading period regulations.

5 Policy recommendations on firm incentives

Finding a fixed blackout window that works across all firm insiders is practically impossible. Furthermore, as we discuss in Section 5.2, the ESO incentive has tolerance effects in which allocating additional ESOs might render the blackout too short. Therefore, it is critical for firms to develop effective incentivizing schemes. We suggest re-examining reload stock options.

5.1 Effective blackout trading periods, too good to be true

The following are our concerns regarding effective blackouts. First, an effective blackout, i.e., $\mathfrak{B} \in (\mathfrak{B}^L, \mathfrak{B}^U]$, might not always exist as $\mathfrak{B}^L > \mathfrak{B}^U$.[12] Second, the boundaries of effective blackouts

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[11] That is, correlated with at least one other firm, while it may or may not be correlated with the market return (systematic).

[12] In that case, no blackout trading period exists that is both adequate and fair.

vary across individuals; hence, there is no uniform effective blackout. Third, even for a particular executive, the effective blackout is not static. Because an executive's total wealth and portfolio constraints change dynamically, a fixed blackout window for a particular insider might switch between different states (inadequate, effective, and excessive) from time to time. Fourth, job termination could reduce the portfolio holding period and, equivalently, extend the blackout trading period from inadequate to effective. Thus, ESOs provide differing incentives for different insider executives, depending, jointly, on the adequacy of blackouts and foreseeability of job termination.

For these four reasons and because, in practice, firms can mandate only a single predetermined blackout to all corporate insiders, developing alternative incentives to ESOs is critical.

5.2 *Tolerance effect of executive stock options*

So far, we have assumed that executives have insights but are incapable of affecting the future risk source. In this case, an ESO would provide a short-term incentive and motivate insider executives to boost the current spot price to achieve a higher derived utility. We have shown that an inadequate blackout invalidates the incentives of firm-granted conventional ESOs.

Now we assume that executives can determine or at least influence the future risk source. The firm then has the motivation to grant more ESOs to better align the interests of executives and shareholders in the long run. We show that, even if the blackout as initially set is adequate, it can become inadequate as the firm grants more ESOs, invalidating both the long-term and short-term incentives of ESOs. [13] We call it the *tolerance effect* of the ESO and offer a scheme for granting RSOs written on the firm's stock as an alternative long-term incentive.

In particular, an insider's unconstrained optimal portfolio is $\pi^{\mathbb{G}*} = [\sigma^{\top}(t)\sigma(t)]^{-1}[b(t) - r(t)\mathbf{1}] + \sigma(t)^{-1}a(t)$. By construction, the noisy information is not a traded asset; that is, ${\pi_{d+1}}^{\mathbb{G}*} = 0$ and only ${\pi_1}^{\mathbb{G}*}$ and the proportion assigned to the risk-free asset $\left(1 - \sum_{i=1}^{d} {\pi_i}^{\mathbb{G}*}\right)$ are affected by $a_1(t)$, which is an increasing function of $W_1(T^*)$. Hence, the higher the terminal value of the firm's stock, the greater the proportion of the firm's stock the executives should optimally hold. If the terminal value of the firm's stock is low enough to make ${\pi_1}^{\mathbb{G}*}$ negative, then the non-hedgeable (i.e., no short selling) constraint prevents the executives from trading optimally. As a result, the executives have the motivation to boost the future terminal value of their stock, i.e., $S_1(T^*)$ [or, equivalently, $W_1(T^*)$] to make the optimal firm stock proportion ${\pi_1}^{\mathbb{G}*}$ positive to rid themselves of the constraint, to get more wealth, and to improve the expected derived utility.

Building on our analysis, we further claim that firms have the motivation to provide stronger

[13] We measure the short-term incentive of an ESO using the first-order derivative of $\mathbb{J}^{\mathbb{G}}$ with respect to $S_1(t)$, and measure the long-term incentive of an ESO using the first-order derivative of $\mathbb{J}^{\mathbb{G}}$ with respect to $S_1(T^*)$. We do not use the ESO's subjective price sensitivity because, with non-transferable non-hedgeable constraints, the objective of the optimal exercise policy and the portfolio optimization problem is to maximize expected utility generated by terminal total wealth rather than to maximize the subjective price of the ESO. Those two coincide only when the portfolio is unconstrained.

incentives to tighten the non-transferable non-hedgeable constraints. If the constraint is stricter, e.g., a large grant of non-transferable non-hedgeable ESOs, then the opportunity set of ${\pi_1}^{\mathbb{G}}$ is $[\zeta, \infty)$, where ζ is a positive constant. Then, the insider executives are motivated to boost $S_1(T^*)$ until the optimal portfolio satisfies, ${\pi_1}^{\mathbb{G}*} \geq \zeta$, which enables them to escape from the non-transferable non-hedgeable constraints. In other words, a stronger incentive comes from stricter non-transferable non-hedgeable constraints by granting more ESOs.

Proposition 2 demonstrates that the lower bound of a blackout, which prevents the executives from exercising all the ESOs immediately after their vesting period, is an increasing function of n, the total number of ESOs initially granted. [This is because $X^{*,\dot{n}}(t_{vo})$ is an increasing function of n.] Even though the firm has set an adequate blackout, such a blackout can become inadequate as the firm grants more ESOs to the executives later. We call this phenomenon that a stronger incentive (i.e., more ESOs) brings stronger non-transferable non-hedgeable constraints that counteract the incentives, the *tolerance effect* of ESOs, which is brought about by the executives' insider trading.

5.3 Reload stock options incentives

In this section, we study the exercise policy and pricing of RSOs. We find that the exercise of American ESOs (RSOs with infinite reloads) is determined backwardly (forwardly) and is affected by (robust to) insider trading and portfolio constraints. Granting ESOs successively induces executives' successive short-term performance, which can be weakened because of the tolerance effect caused by the insider trading. Granting long-term RSOs with infinite reloads incentivizes insider executives' long-term performance. We recommend that firms reconsider using RSOs, which have been fallen out of favor in recent years.

An RSO, invented by Frederic W. Cook and Co. in 1987, is a non-transferable non-hedgeable American call option that grants additional at-the-money options upon exercising the initial one. The option holder pays the strike price in stock already possessed instead of paying in cash (stock-for-stock). Meanwhile, a new strike is set to be the market value of the underlying stock at the time the option is exercised.

Dybvig and Loewenstein (2003)[14] made a breakthrough contribution on RSO pricing. They showed that the optimal exercise policy of a RSO with infinite reloads is to exercise the options whenever the stock price reaches a historical record new high, and the value of the reload option always lies between the value of an American call and the stock price, irrespective of the number of reloads and the maturity.

We claim that if we further consider executives' insider trading as well as non-transferable non-hedgeable portfolio constraints, the optimal exercise policy stated in Dybvig and Loewenstein (2003) still holds. The logic is as follows: Cvitanić and Karatzas (1992) and Karatzas and Kou (1996) elegantly

[14] Saly et al. (1999), Brenner et al. (2000), Dai and Kwok (2005), Ingersoll (2007), Bélanger and Forsyth (2008), Dai and Kwok (2008) have also contributed to the RSO pricing literature.

transform a constrained portfolio optimization problem into an unconstrained one with adjusted drift rates; Pikovsky and Karatzas (1996) endow the portfolio optimization framework the flexibility to incorporate the anticipative feature of insider trading, again through drift rate adjustment; however, whether the results in Dybvig and Loewenstein (2003) hold does not depend on the value of drifts as inputs; therefore, considering executives' insider trading and portfolio constraints, the optimal exercise policy for RSO with infinite reloads holds.

We now assume that (i) executives have insider information regarding the terminal values of a driving Brownian motion; (ii) executives pay the strike price in mature stock already in their possession, rather than in cash; (iii) executives are always holding enough mature shares to pay for the exercise price (we do not assume the employee can borrow the necessary shares); (iv) a RSO can be exercised only after its predetermined vesting period; (v) executives are prohibited from short selling the firm's stock and transferring the options; and (vi) executives are not allowed to sell the shares they own during the blackout trading period. Note that exercising a RSO through "stock-for-stock" belongs to the intra-company approach defined in footnote (9); it does not involve contemporaneous sale into the market and, hence, is allowed during blackout (See Nathan and Hoffman 2013).

Holding RSOs and reloading them with an optimal reload policy is equivalent to accumulating the instant payoff at the time of each exercise, realized at any appropriate reloading time. Therefore, without considering the present value discount, the realized cash payoff at time t is the difference between the current value and the previous historical record highs [denoted as $d\Lambda(t)$], where $\Lambda(t)$, the historical high, is a non-decreasing envelope forwardly created, See Figure 1.

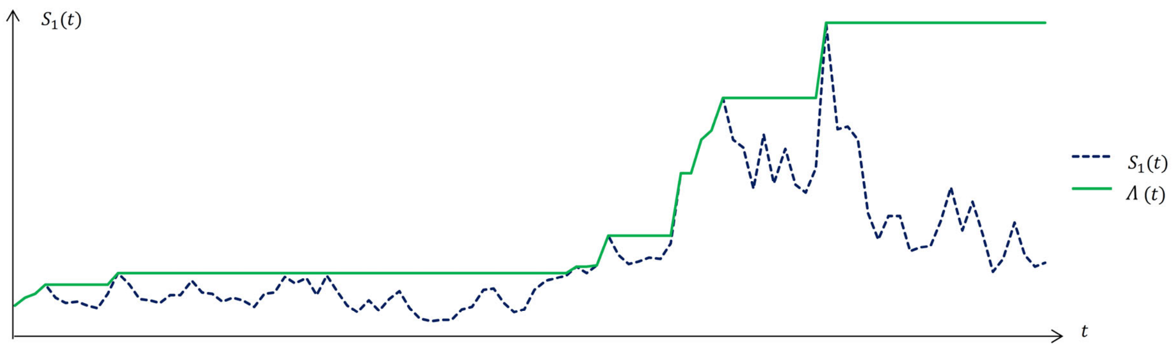

Figure 1

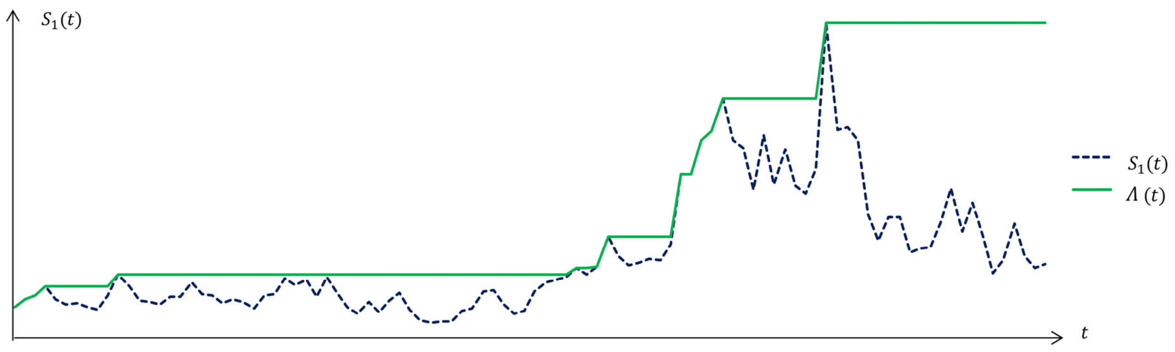

Figure 1. RSO exercise policy envelope. As the number of reload opportunities approaches infinity, its randomness, in terms of the optimal reload policy, is weakened and the certainty is enhanced. The reload time is still determined by the firm stock price; hence, it is still a stopping time. However, determining the reload time is not a free boundary problem anymore. The reload option is more like a barrier option, and the pricing is much easier.

We employ the common practice of setting the number of additional non-transferable non-hedgeable American at-the-money call options, granted upon exercising the initial one, equal to the pre- and post-exercise strike price ratio. At any time $u \in [t, T]$, the number of RSOs converted from one RSO at time t^0 is $S_1(t^0)/S_1(u)$, where t^0 is the RSO granting time at which the strike price of the RSO was initially set as $S_1(t^0)$.

To price RSOs, we refer to a result of Dybvig and Loewenstein (2003, Lemma 1, p. 9). However, we use a different stochastic discount factor, namely, an insider's: $\{\mathcal{G}_t\}$-progressively measurable subjective stochastic discount factor accounting for non-transferable non-hedgeable constraints as follows. For any time $t \in [t^0, T]$,

$$H_v^{\mathbb{G}}(t) \triangleq \exp\left\{-\int_0^t r_v^{\mathbb{G}}(s)\, ds\right\} Z_v^{\mathbb{G}}(t), \tag{23}$$

where

$$Z_v^{\mathbb{G}}(t) \triangleq \exp\left\{-\int_0^t [\theta_{a,v}(s)]^{\mathsf{T}} d\widetilde{W}(s) - \frac{1}{2}\int_0^t \|\theta_{a,v}(s)\|^2 ds\right\}. \tag{24}$$

Processes $r_v^{\mathbb{G}}$ and $\theta_{a,v}$ are defined in Section 3.6.

An insider's subjective price of an at-the-money RSO, expiring at T, with infinite reloads, which was converted from one RSO granted at time t^0, is

$$\hat{p}(S_1(t), t^0, t, T) = \mathbb{E}^{\mathbb{P}}\left[\int_t^T \frac{H_v^{\mathbb{G}}(u)}{H_v^{\mathbb{G}}(t)} \frac{S_1(t^0)}{S_1(u)} \left(dS_1(u)\right)^+ \mathbf{1}_{\{S_1(u)=\Lambda(u)\}} \middle| \mathcal{F}_t\right], \tag{25}$$

where $\Lambda(u) \equiv \max(S_1(s), 0 \le s \le u)$ is the firm's stock price's envelope.

The subjective stochastic discount factor process for insiders, $H_v^{\mathbb{G}}$, is determined by a, the information compensating process, as well as $v^{\mathbb{G}}$, the drift rate adjustment reflecting insiders' portfolio constraints. Setting $v^{\mathbb{G}} \equiv 0$, we obtain the objective stochastic discount factor, which in turn determines the firm's cost of a RSO granted to insiders, denoted as $\tilde{p}(S_1(t), t^0, t, T)$.

According to the Law of One Price, using a different discount factor does not vary the upper bound of options. Therefore, after we consider insider information and portfolio constraints, the result in Dybvig and Loewenstein (2003) still holds, that the upper bound, at any time $\boldsymbol{t}$, of the firm's cost (objective price) of granting an at-the-money RSO with infinite reloads is the spot stock price at the initial granting time.

RSOs fell out of favor around 2006, and firms gradually stopped granting RSOs thereafter for two reasons: the pricing difficulty and the claim that RSOs bestow too many lucrative shares to executives.

First, in 2004, the Financial Accounting Standard Board made the RSO optional reporting mandatory [FAS123(R)]. It reacted to the extensive use of share-based compensation, asking for reports of fair value, reflecting grant-date share price and other pertinent factors, including volatility, restrictions, and inherent conditions.

Although much progress has been made on ESO pricing, considering non-transferable non-hedgeable constraints and adding the reload feature escalates the pricing difficulty, the Board continues to believe that the reload term makes it impossible to estimate a reasonably fair value of options at the grant date. It states that subsequent granting of reload options should be accounted for as a separate award when the reload options are granted [See FAS123(R) paragraphs 24 to 26; see also Saly et al. 1999]. However, the objective price, $\tilde{p}(S_1(t), t^0, t, T)$ (the case where we set $v^{\mathbb{G}} \equiv 0$) gives the firm's cost of a RSO, taking all the aforementioned factors into account.

Second, RSOs have been blamed for bestowing too many lucrative shares to the executives. Our work endeavors to test the truth or falsehood of that claim from a new perspective by taking executives' insider trading into account. The objective price of the RSO, $\tilde{p}(S_1(t), t^0, t, T)$, is certainly less than $S_1(t)$, which shows that the firm's cost of granting one at-the-money RSO with infinite reloads is no more than granting one share of firm stock. Hence, the claim that RSOs are money pumps for executives is groundless.

6 Simulation and sensitivity analysis

We use Monte Carlo simulation to demonstrate how executives' insider information changes the incentivizing mechanism of their ESOs. For simplicity, we consider European ESOs. We study two volatility regimes: a low one (Table 1) and a high one (Table 2). For each volatility regime, we study two types of insider information: good news (Panel B) and bad news (Panel C), and compare the results with those of an outsider executive, who has no information (Panel A).

For each panel, we report three ESO prices: "plain vanilla" Black-Scholes prices, ESO objective prices, and executives' subjective prices. The objective price (also termed as the firm cost in the literature) is the price without considering portfolio constraints, while subjective price is the one taking non-transferable non-hedgeable constraints into account. For outsider executives, the objective price is the Black-Scholes price. However, for insider executives, they are not equivalent. See Panel A in Table 1 and Table 2.

We distinguish between and report utility incentives and price incentives. We define *utility incentives* as the change in executive's derived utilities per unit of stock price change, i.e., $\partial J^{\mathbb{F}}\left(x_t^{\mathbb{F}}, t, T\right)/\partial S_1(t)$ for outsider executives and $\partial J^{\mathbb{G}}\left(x_t^{\mathbb{G}}, t, T\right)/\partial S_1(t)$ for insiders.

We define *price incentives* as the delta of the subjective price $\hat{p}(t)$, i.e., the change in the logarithm of the subjective price of an ESO per unit of share price change, $\partial ln(\hat{p}(t))/\partial S_1(t)$.

We also present the *deadweight cost* of granting ESOs, which is an ESO's objective price net of the subjective price; that is, $\tilde{p}(t) - \hat{p}(t)$. A positive (negative) value of deadweight cost indicates an ESO granting efficiency loss (gain).

We measure and report the overall ESO *granting efficiency* as the utility incentives adjusted by multiplying a deadweight cost discount (premium), $e^{-\left[\frac{\tilde{p}(t)-\hat{p}(t)}{\tilde{p}(t)}\right]}$, if the deadweight cost is positive (negative); that is, we define the granting efficiency by $e^{-\left[\frac{\tilde{p}(t)-\hat{p}(t)}{\tilde{p}(t)}\right]} \partial J^{\mathbb{G}}\left(x_t^{\mathbb{G}}, t, T\right)/\partial S_1(t)$. We argue that the utility incentives do not take into account the fact that the dead weight cost is not always positive, and if it is negative, it is a good thing. This is why we multiply the utility incentive by this exponential term, to discount the utility incentive if the deadweight cost ratio is positive and to increase it if the deadweight cost ratio is negative, much like taking a present value.

Our simulated results reconcile with our theoretical results. The subjective prices of ESOs perceived by outsider executives are less than or equal to the firms' granting costs, which implies positive[15] deadweight costs of ESOs granted to outsider executives and which, in turn, reduces ESO incentivizing and results in a lower granting efficiency. By contrast, the subjective price perceived by insider executives is usually greater than firms' granting costs, resulting in a negative deadweight cost for ESOs granted to insider executives, which increases the ESO incentivizing. However, when executives have insider information, their utility incentives could become weaker; therefore, the overall granting efficiency of ESOs to insider executives becomes lower than to outsider executives. This is often the case in simulations of low-volatility regimes regardless of whether insiders' information is good news or bad news (Table 1, Panel A, B, C) and with high-volatility regimes when insiders' information is good news (see Table 2, Panel A, B). Of 87 scenarios, 64 show a deficiency of granting ESOs due to insider executives' predictive information. It is remarkable that 25 of 29 scenarios in the high-volatility regime when insiders' information is good news demonstrate an ESO granting deficiency (i.e., the granting efficiency is negative). However, under high-volatility regimes when insiders' news is bad, the granting efficiency to insider executives is higher than to outsider executives most of the time.

We create scenarios to illustrate how parameter changes affect ESO incentives. We study both derived utility sensitivity and ESO subjective price sensitivity to changes in the spot stock price, the market index, the correlation between these two, stock drifts, the index drift, the risk-free rate,

[15] The occasional negative deadweight cost observed is due to rounding errors.

investment horizons, stock volatilities, index volatility, ESO strike prices, vesting periods of restricted stocks, initial endowments (number of shares) of nonvested stocks, initial endowments (number of units) of non-transferable non-hedgeable ESOs, and initial cash endowments in dollars.

The following comparative statics are of high interest to executives, firms, and the public.

In Table 1, we present results under the low-volatility regime: 20% stock volatility, 10% index volatility. In Table 2, we present results under the high-volatility regime: 50% stock volatility, 30% index volatility).

1. ESOs incentivize executives to increase underlying stock prices. In most cases, these incentives, as measured by utilities' sensitivities to stock price changes, are stronger to the insider executives with bad news information, than to outsider executives. The incentives to insider executives with good news information are weaker than the incentives to outsider executives.
2. For the low-volatility regime, symmetric impacts dominate: insider information, whether good or bad news, increases insiders' objective prices and subjective prices of their ESOs, compared to outsiders'. However, prices increase more to insider executives with good news. For high-volatility regimes, asymmetric impacts dominate, meaning that if insiders' information is good (bad) news, it increases (decreases) insider executives' objective and subjective prices of their ESOs, compared with outsider executives' prices.
3. Objective prices are usually higher (lower) than outsider (insider) executives' subjective prices. Usually objective and subjective prices move in tandem when parameters change. Exceptions occur when risk structures (stock volatility, index volatility, index-stock correlation) change.
4. It is well known, that ESOs incentivize executives to increase firm's stock volatility. We find that for executives with bad news insider information, regardless of the volatility regime, there are inverse relationships between stock volatilities and ESO subjective prices. See Panel C. (h) in Table 1 and 2.
5. A lower positive correlation between firms' stock returns and index returns result in higher positive granting efficiencies to outsider executives, regardless of the volatility regime. This feature holds for insider executives only under the high-volatility regime and good news insider executives' information. See Panel A. (c) in Table 1 and 2 and Panel B. (c) in Table 2.
6. We assume that ESOs expire at the end of the investment horizon. Longer option maturities add values to both subjective and objective prices. Longer investment horizons, being proportional to information disclosure times, at a fixed rate (e.g., 5/6), do not necessarily improve granting efficiencies. See Panel B. (g) and Panel C. (g) in Table 1 and 2.
7. We fixed the investment horizon at five years and simulated different information disclosure times. We find that the closer investment horizons are to information disclosure times, the higher are incentives' efficiencies, suggesting that although blackouts are of critical importance, blackouts should not be too long. See Panel B. (o) and Panel C. (o) in Table 1 and 2.
8. While stocks' drift rates are not part of the Black-Scholes pricing formula and, thus, do not

affect the objective prices of outsider executives' European ESOs, they affect outsider executives' subjective prices. Higher stock drift rates do not necessarily result in higher outsiders' subjective prices, see Panel A. (d) in Table 1. However, higher stock drift rates result in greater insiders' subjective prices, regardless of the volatility regime and whether insiders' information is good or bad news. See Panel B. (d) and Panel C. (d) in Table 1 and 2.

9. Subjective and objective ESO prices are affected also by the drift rate and volatility of the market index. As the index drift rate increases, outsider executives' objective and subjective prices decrease. Similarly, as the index's volatility decreases, these prices tend to decrease. Intuitively, as the index becomes more attractive, i.e., as its drift rate increases or its volatility decreases (all else being equal), the ESO's position becomes relatively less attractive. However, if executives have insider information on firms' future stock returns, the relationship between stocks and market index changes from substitutes to complements, that is, insiders' subjective price of ESOs increases as the index's drift increases and as the index's volatility decreases. See Panel A-C. (e, i) in Table 1 and 2.

10. To outsider executives, index drift rate and granting efficiency move in tandem. To insider executives, index drift rate and granting efficiency move in tandem in the low-volatility regime but move in different directions in the high-volatility regime. See Panel A-C. (e) in Table 1 and 2.

11. In the low (high) volatility regime, low (high) index volatilities induce high granting efficiencies. See Panel A-C. (i) in Table 1 and 2.

12. In the low-volatility regime, the risk-free rate and insider information characteristics jointly have an impact on granting efficiencies. In particular, very low risk-free rates induce negative granting efficiency if insider executives have good news information, while very high risk-free rates induce negative granting efficiencies if insider executives have bad news. See Panel B-C. (f) in Table 1.

13. The impact of ESO moneyness on granting efficiency depends on the volatility regime. In low-volatility regimes, in-the-money ESOs have the highest granting efficiency to outsider executives. However, to insider executives with good news information, out-of-the-money ESOs have the highest granting efficiency. Firms should grant at-the-money ESOs to executives with bad news insider information and stop granting them with out-of-the-money ESOs due to negative granting efficiency. In high-volatility regimes, out-of-the-money ESOs have the highest granting efficiency to outsider executives. For insider executives who have good news, both in-the-money and out-of-the-money ESOs have higher granting efficiency than at-the-money ESOs. However, granting at-the-money ESOs to insider executives who have bad news information is most efficient; but other granting schemes, such as granting in-the-money or out-of-the-money ESOs, do not result in negative granting efficiency. See Panel A-C. (j) in Table 1 and 2.

14. For insiders, excessively long stock vesting periods usually induce a low granting efficiency, because insiders cannot utilize the insider information. See Panel B-C. (k) in Table 1 and 2.
15. The greater the ratios of option endowments values over total initial wealth (options, stocks, and cash), the greater are granting efficiencies, thus, the stronger is ESO alignment of executive and shareholders' interests. However, granting excessive number of ESOs to insider executives, harms incentivization and results in a negative granting efficiency. See Panel A-C. (l) in Table 1 and 2.
16. Granting efficiency, at any level of initial restricted stock endowment, is unconditionally positive. However, we observe that the lowest level of initial restricted stock endowment induces the highest granting efficiency. See Panel A-C. (m) in Table 1 and 2.
17. A large cash endowment is important to incentivize insider executives only when they have good news information in the low-volatility regime. See Panel A-C. (n) in Table 1 and 2.
    In the low (high) volatility regime, insider executives with information that is mildly (extremely) good news with high precision face the highest granting efficiency. However, if the information is very bad news, the ESO incentives disappear and granting efficiency could be negative, regardless of the volatility regime. See Panel B-C. (p, q, r) in Table 1 and 2.

7 Conclusion

Properly incentivizing executives is essential for firms' performance, economic growth, and societal welfare. The predominant executives' incentivizing instrument has been non-transferable non-hedgeable American executive stock options. Colwell et al. (2015) were the first to analytically price such options in general. We demonstrate that executives' insider information nullifies conventional ESO incentives. Despite non-transferable non-hedgeable restrictions and insider trading restrictions imposed in Securities Exchange Act of 1934, Rule 10b5, executives may use portfolio optimization-style trading, rather than arbitrage style, in their outside wealth portfolios. The reason is that Securities Exchange Act of 1934, Rule 10b5 Rule 10b5-1, which allows trading according to premeditated plans before arrival of insider information, cannot be generally enforced.

We show how granting insider executives with infinite reload of non-transferable non-hedgeable American ESOs combined with blackout trading periods may realign executives' and stockholders' interests.

Analytically, we price RSOs for insider executives and identify their optimal exercise policies. We identify lower and upper bounds of blackout trading periods. Exceeding lower bounds results in complete liquidation of ESOs, and exceeding upper bounds reduces executives' derived utility to below that of corresponding outsiders.

We adopt constrained primary asset portfolio optimization techniques and combine them with enlarged filtration techniques, which we further develop to allow for insiders' noisy information. To facilitate the pricing of insiders' information, we introduce imaginary non-tradable assets.

Our Monte Carlo simulation confirms that insider information could weaken ESO incentivizing power. The weakening extent depends on volatility regimes and insider news type (good/bad). There is stronger weakening under low volatility regimes and under bad news. Sensitivity analyses agree with our theoretical results.

Policy implications suggest the reintroduction of the out-of-favor RSO combined with firm-imposed and SEC-regulated blackout trading periods of firms' issued securities. When insider information is idiosyncratic (systematic), a blackout trading prohibition includes the firm's stock only (all assets).

Future empirical research will test the implications of this paper, and future theoretical research will address the issues here with the added assumption of random job termination for executives.

Table 1

Sensitivity analysis and determinants of ESO efficiency – Low-volatility regime (20% stock volatility, 10% index volatility)

Panel A: Outsider Executives' Incentive of ESO

		BS.	Obj.	Sub.	Inc_U	Inc_P	D.W.C	Eff.
	Benchmark scenario	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
(a)	stock price = 12	4.118	4.118	4.117	2.4e-05	0.400	2.3e-04	2.4e-05
	stock price = 10	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	stock price = 6	0.384	0.384	0.384	0.0e+00	1.415	6.5e-06	0.0e+00
(b)	index price = 10	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	index price = 6	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	index price = 2	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
(c)	correlation = 0.9	2.522	2.522	2.522	0.0e+00	0.558	-1.3e-05	0.0e+00
	correlation = 0.6	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	correlation = 0.3	2.522	2.522	2.522	5.1e-04	0.550	8.4e-05	5.1e-04
(d)	stock drift = 0.2	2.522	2.522	2.513	2.0e-03	0.560	8.9e-03	2.0e-03
	stock drift = 0.15	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	stock drift = 0.12	2.522	2.522	2.519	4.7e-04	0.559	2.7e-03	4.7e-04
(e)	index drift = 0.12	2.522	2.522	2.123	3.5e-03	0.512	4.0e-01	3.0e-03
	index drift = 0.08	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	index drift = 0.06	2.522	2.522	2.522	1.4e-06	0.557	-1.4e-05	1.4e-06
(f)	risk-free rate = 0.06	3.000	3.000	3.000	0.0e+00	0.511	0.0e+00	0.0e+00
	risk-free rate = 0.04	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	risk-free rate = 0.02	2.069	2.069	2.065	2.3e-03	0.596	4.5e-03	2.2e-03
(g)	investment horizon = 7	3.140	3.140	3.140	4.8e-05	0.475	-5.0e-06	4.8e-05
	investment horizon = 5	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	investment horizon = 3	1.801	1.801	1.801	1.9e-07	0.719	-2.0e-06	1.9e-07
(h)	stock volatility = 0.5	4.662	4.662	2.846	1.4e-02	0.354	1.8e+00	9.5e-03
	stock volatility = 0.2	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	stock volatility = 0.13	2.036	2.036	1.993	1.9e-03	0.700	4.3e-02	1.8e-03
(i)	index volatility = 0.15	2.522	2.522	2.522	0.0e+00	0.555	3.9e-05	0.0e+00
	index volatility = 0.1	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	index volatility = 0.05	2.522	2.522	2.110	1.2e-02	0.532	4.1e-01	1.0e-02
(j)	strike = 12	1.724	1.724	1.724	4.5e-06	0.657	4.2e-05	4.5e-06
	strike = 10	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	strike = 8	3.604	3.604	3.604	2.6e-05	0.465	2.2e-04	2.6e-05
(k)	stock vesting period = 1.5	2.670	2.670	2.670	0.0e+00	0.000	1.7e-04	0.0e+00
	stock vesting period = 1	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	stock vesting period = 0.5	2.522	2.522	2.522	0.0e+00	0.553	3.5e-06	0.0e+00
(l)	option granted =2000 shares	2.522	2.522	2.488	1.5e-02	0.553	3.4e-02	1.5e-02
	option granted = 200 shares	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	option granted = 20 shares	2.522	2.522	2.522	1.3e-06	0.553	5.4e-05	1.3e-06
(m)	stock granted = 2000 shares	2.522	2.522	2.522	5.9e-06	0.553	1.0e-04	5.9e-06
	stock granted = 200 shares	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	Stock granted = 20 shares	2.522	2.522	2.522	5.8e-04	0.553	2.7e-04	5.8e-04
(n)	cash endowment = 10000	2.522	2.522	2.522	0.0e+00	0.553	0.0e+00	0.0e+00
	cash endowment = 1000	2.522	2.522	2.522	8.7e-06	0.553	5.5e-05	8.7e-06
	cash endowment = 500	2.522	2.522	2.522	2.4e-05	0.553	7.6e-05	2.4e-05

Panel B: Insider Executives' Incentive of ESO with insider information as good news ($W_1(T^*) = 1, W_{d+1}(T^*) = 0.5$)

		BS.	Obj.	Sub.	Inc_U	Inc_P	D.W.C	Eff.
	Benchmark scenario	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	stock price = 12	4.118	23.617	23.906	5.9e-05	0.111	-0.289	6.0e-05
(a)	stock price = 10	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	stock price = 6	0.384	7.817	7.914	1.8e-04	0.337	-0.097	1.9e-04
	index price = 10	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
(b)	index price = 6	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	index price = 2	2.522	18.350	18.576	4.8e-05	0.143	-0.228	4.9e-05
	correlation = 0.9	2.522	18.684	18.699	3.8e-06	0.143	-0.014	3.8e-06
(c)	correlation = 0.6	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	correlation = 0.3	2.522	18.241	18.548	-1.6e-04	0.144	-0.307	-1.6e-04
	stock drift = 0.2	2.522	28.295	28.346	9.7e-05	0.129	-0.051	9.7e-05
(d)	stock drift = 0.15	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	stock drift = 0.12	2.522	13.904	13.913	-5.7e-04	0.157	-0.010	-5.8e-04
	index drift = 0.12	2.522	18.374	19.835	1.2e-03	0.169	-1.461	1.3e-03
(e)	index drift = 0.08	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	index drift = 0.06	2.522	18.625	18.858	2.5e-05	0.143	-0.233	2.5e-05
	risk-free rate = 0.06	3.000	16.203	16.347	3.1e-05	0.145	-0.144	3.1e-05
(f)	risk-free rate = 0.04	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	risk-free rate = 0.02	2.069	20.680	21.145	-4.2e-04	0.146	-0.465	-4.3e-04
	investment horizon = 7	3.140	26.127	26.159	2.6e-05	0.129	-0.031	2.6e-05
(g)	investment horizon = 5	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	investment horizon = 3	1.801	12.004	12.047	5.2e-05	0.172	-0.043	5.2e-05
	stock volatility = 0.5	4.662	24.158	27.564	3.9e-03	0.140	-3.406	4.5e-03
(h)	stock volatility = 0.2	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	stock volatility = 0.13	2.036	16.316	16.356	1.7e-04	0.151	-0.040	1.7e-04
	index volatility = 0.15	2.522	18.530	18.770	3.3e-05	0.143	-0.240	3.3e-05
(i)	index volatility = 0.1	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	index volatility = 0.05	2.522	18.374	20.740	1.7e-03	0.174	-2.365	1.9e-03
	strike = 12	1.724	16.753	16.967	1.7e-04	0.157	-0.214	1.7e-04
(j)	strike = 10	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	strike = 8	3.604	19.947	20.182	1.7e-05	0.132	-0.234	1.7e-05
	stock vesting period = 1.5	2.670	18.350	18.576	0.0e+00	0.000	-0.226	0.0e+00
(k)	stock vesting period = 1	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	stock vesting period = 0.5	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	option granted =2000 shares	2.522	18.350	18.605	-3.9e-03	0.142	-0.255	-4.0e-03
(l)	option granted = 200 shares	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	option granted = 20 shares	2.522	18.350	18.538	3.1e-05	0.143	-0.188	3.1e-05
	stock granted = 2000 shares	2.522	18.350	18.544	3.5e-06	0.143	-0.194	3.5e-06
(m)	stock granted = 200 shares	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	Stock granted = 20 shares	2.522	18.350	18.618	1.5e-04	0.143	-0.268	1.5e-04
	cash endowment = 10000	2.522	18.350	18.549	1.9e-04	0.143	-0.199	1.9e-04
(n)	cash endowment = 1000	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	cash endowment = 500	2.522	18.350	18.589	1.6e-04	0.143	-0.239	1.6e-04
	info disclosure time = 6.4 yr	2.522	13.595	13.598	2.9e-05	0.159	-0.002	2.9e-05
(o)	info disclosure time = 6 yr	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	info disclosure time = 5.6 yr	2.522	30.535	32.111	6.5e-05	0.126	-1.575	6.8e-05
	information= 1.5	2.522	23.792	23.951	1.4e-05	0.134	-0.159	1.4e-05
(p)	information = 1	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	information = 0.2	2.522	11.514	11.852	3.2e-04	0.169	-0.339	3.3e-04
	noise = 0.8	2.522	18.350	18.560	3.4e-05	0.143	-0.210	3.5e-05
(q)	noise = 0.5	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	noise = 0.3	2.522	18.350	18.592	5.5e-05	0.143	-0.242	5.6e-05
	info quality = 0.9	2.522	18.350	19.120	1.0e-03	0.144	-0.770	1.1e-03
(r)	info quality = 0.6	2.522	18.350	18.578	4.8e-05	0.143	-0.228	4.9e-05
	info quality = 0.2	2.522	18.350	18.327	7.4e-06	0.144	0.023	7.4e-06

Panel C: Insider Executives' Incentive of ESO with insider information as bad news ($W_1(T^*) = 1, W_{d+1}(T^*) = 0.5$)

		BS.	Obj.	Sub.	Inc_U	Inc_P	D.W.C	Eff.
	Benchmark scenario	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	stock price = 12	4.118	6.926	8.003	-3.8e-03	0.182	-1.077	-4.5e-03
(a)	stock price = 10	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	stock price = 6	0.384	0.505	0.569	6.9e-03	1.074	-0.064	7.9e-03
	index price = 10	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
(b)	index price = 6	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	index price = 2	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	correlation = 0.9	2.522	4.569	4.676	2.4e-03	0.269	-0.107	2.4e-03
(c)	correlation = 0.6	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	correlation = 0.3	2.522	4.448	5.383	2.0e-04	0.281	-0.935	2.4e-04
	stock drift = 0.2	2.522	9.110	10.594	-6.5e-06	0.202	-1.484	-7.6e-06
(d)	stock drift = 0.15	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	stock drift = 0.12	2.522	2.492	2.792	-8.9e-03	0.372	-0.300	-1.0e-02
	index drift = 0.12	2.522	4.492	8.308	1.5e-01	-0.002	-3.816	3.4e-01
(e)	index drift = 0.08	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	index drift = 0.06	2.522	4.549	5.138	-7.0e-03	0.272	-0.589	-8.0e-03
	risk-free rate = 0.06	3.000	3.819	4.282	-2.5e-03	0.281	-0.463	-2.8e-03
(f)	risk-free rate = 0.04	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	risk-free rate = 0.02	2.069	5.198	6.106	1.9e-03	0.267	-0.907	2.3e-03
	investment horizon = 7	3.140	8.464	8.443	6.0e-04	0.191	0.021	5.9e-04
(g)	investment horizon = 5	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	investment horizon = 3	1.801	1.344	1.548	1.4e-02	0.560	-0.204	1.6e-02
	stock volatility = 0.5	4.662	0.421	0.798	1.4e-02	0.374	-0.377	3.4e-02
(h)	stock volatility = 0.2	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	stock volatility = 0.13	2.036	6.897	7.922	9.4e-05	0.229	-1.025	1.1e-04
	index volatility = 0.15	2.522	4.519	5.135	1.7e-03	0.272	-0.615	1.9e-03
(i)	index volatility = 0.1	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	index volatility = 0.05	2.522	4.492	6.884	2.2e-01	-0.448	-2.392	3.7e-01
	strike = 12	1.724	3.033	3.473	-1.7e-03	0.367	-0.441	-1.9e-03
(j)	strike = 10	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	strike = 8	3.604	6.036	6.951	3.4e-03	0.209	-0.914	3.9e-03
	stock vesting period = 1.5	2.670	4.471	5.143	0.0e+00	0.000	-0.672	0.0e+00
(k)	stock vesting period = 1	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	stock vesting period = 0.5	2.522	4.471	5.147	8.0e-03	0.280	-0.676	9.3e-03
	option granted =2000 shares	2.522	4.471	6.221	-7.3e-02	0.289	-1.750	-1.1e-01
(l)	option granted = 200 shares	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	option granted = 20 shares	2.522	4.471	4.941	-1.4e-03	0.270	-0.469	-1.6e-03
	stock granted = 2000 shares	2.522	4.471	4.962	4.8e-03	0.269	-0.491	5.4e-03
(m)	stock granted = 200 shares	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	Stock granted = 20 shares	2.522	4.471	5.428	1.7e-02	0.327	-0.956	2.1e-02
	cash endowment = 10000	2.522	4.471	4.985	4.5e-03	0.270	-0.513	5.0e-03
(n)	cash endowment = 1000	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	cash endowment = 500	2.522	4.471	5.201	9.4e-03	0.277	-0.730	1.1e-02
	info disclosure time = 6.4 yr	2.522	3.516	3.652	6.4e-04	0.322	-0.136	6.6e-04
(o)	info disclosure time = 6 yr	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	info disclosure time = 5.6 yr	2.522	6.925	9.635	2.0e-02	0.214	-2.710	2.9e-02
	information= -1.5	2.522	2.477	3.055	-1.6e-03	0.372	-0.578	-2.1e-03
(p)	information = -1	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	information = -0.2	2.522	8.794	9.402	7.0e-04	0.191	-0.608	7.5e-04
	noise = -0.8	2.522	4.471	5.222	1.5e-02	0.278	-0.751	1.8e-02
(q)	noise = -0.5	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	noise =- 0.3	2.522	4.471	5.105	1.1e-02	0.272	-0.634	1.3e-02
	info quality = 0.9	2.522	4.471	5.972	5.4e-04	0.268	-1.501	7.5e-04
(r)	info quality = 0.6	2.522	4.471	5.152	7.5e-03	0.279	-0.681	8.8e-03
	info quality = 0.2	2.522	4.471	4.486	1.6e-03	0.272	-0.015	1.6e-03

Table 1 lists the results of ESO incentive sensitivity analysis, namely, how does parameter change affect the change of ESO incentive for a low-volatility regime. Panel A reports the results of outsider executives. Panel B (C) reports the results of insider executives who acknowledge a noisy information as good (bad) news. Each panel displays all the determinants affecting (European) ESO efficiency [Eff.], which is defined as the deadweight-cost-adjusted incentive. The deadweight cost [D.W.C.] is the objective price [Obj.] net of the subjective price [Sub.]. The incentive [Inc_U] is the percentage change of outsider executives' derived utility w.r.t. stock spot price. We also list a price incentive [Inc_P], which is the percentage change of executives' logarithm subjective ESO price w.r.t. stock spot price change. Each subpanel lists three levels (i.e., low, benchmark, high, levels *w.r.t.* the determinant) of results from top to bottom.

Table 2

Sensitivity Analysis and Determinants of ESO Efficiency – High-Volatility regime (50% stock volatility, 30% index volatility)

Panel A: Outsider Executives' Incentive of ESO

		BS.	Obj.	Sub.	Inc_U	Inc_P	D.W.C	Eff.
	Benchmark scenario	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
(a)	stock price = 12	6.241	6.241	5.556	7.7e-03	0.263	6.9e-01	6.9e-03
	stock price = 10	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	stock price = 6	1.895	1.895	1.666	2.1e-02	0.601	2.3e-01	1.9e-02
(b)	index price = 10	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	index price = 6	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	index price = 2	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
(c)	correlation = 0.9	4.662	4.662	4.616	9.8e-04	0.316	4.7e-02	9.7e-04
	correlation = 0.6	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	correlation = 0.3	4.662	4.662	3.819	1.5e-02	0.328	8.4e-01	1.2e-02
(d)	stock drift = 0.2	4.662	4.662	4.568	6.7e-03	0.318	9.4e-02	6.6e-03
	stock drift = 0.15	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	stock drift = 0.12	4.662	4.662	3.652	1.3e-02	0.333	1.0e+00	1.1e-02
(e)	index drift = 0.12	4.662	4.662	3.526	1.3e-02	0.334	1.1e+00	1.0e-02
	index drift = 0.08	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	index drift = 0.06	4.662	4.662	4.275	1.0e-02	0.323	3.9e-01	9.3e-03
(f)	risk-free rate = 0.06	4.944	4.944	4.309	1.1e-02	0.321	6.4e-01	9.4e-03
	risk-free rate = 0.04	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	risk-free rate = 0.02	4.379	4.379	3.876	1.1e-02	0.331	5.0e-01	9.5e-03
(g)	investment horizon = 7	5.459	5.459	4.968	1.1e-02	0.289	4.9e-01	9.8e-03
	investment horizon = 5	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	investment horizon = 3	3.612	3.612	3.177	1.1e-02	0.390	4.3e-01	9.6e-03
(h)	stock volatility = 0.9	6.991	6.991	2.618	8.1e-02	0.242	4.4e+00	4.4e-02
	stock volatility = 0.5	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	stock volatility = 0.4	3.970	3.970	3.909	4.7e-03	0.360	6.1e-02	4.6e-03
(i)	index volatility = 0.4	4.662	4.662	4.217	1.1e-02	0.322	4.5e-01	1.0e-02
	index volatility = 0.3	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	index volatility = 0.1	4.662	4.662	2.846	1.4e-02	0.354	1.8e+00	9.5e-03
(j)	strike = 12	4.147	4.147	3.597	1.2e-02	0.340	5.5e-01	1.0e-02
	strike = 10	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	strike = 8	5.287	5.287	4.759	9.5e-03	0.308	5.3e-01	8.6e-03
(k)	stock vesting period = 1.5	4.816	4.816	4.168	0.0e+00	0.000	6.5e-01	0.0e+00
	stock vesting period = 1	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	stock vesting period = 0.5	4.662	4.662	4.201	7.9e-03	0.324	4.6e-01	7.2e-03
(l)	option granted =2000 shares	4.662	4.662	2.878	8.4e-01	0.376	1.8e+00	5.7e-01
	option granted = 200 shares	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	option granted = 20 shares	4.662	4.662	4.490	5.0e-03	0.318	1.7e-01	4.9e-03
(m)	stock granted = 2000 shares	4.662	4.662	4.418	1.4e-03	0.325	2.4e-01	1.4e-03
	stock granted = 200 shares	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	Stock granted = 20 shares	4.662	4.662	3.607	4.0e-02	0.330	1.1e+00	3.2e-02
(n)	cash endowment = 10000	4.662	4.662	4.654	2.8e-04	0.317	8.2e-03	2.8e-04
	cash endowment = 1000	4.662	4.662	4.114	1.1e-02	0.329	5.5e-01	9.6e-03
	cash endowment = 500	4.662	4.662	3.993	7.6e-03	0.333	6.7e-01	6.6e-03

Panel B: Insider Executives' Incentive of ESO with insider information as good news ($W_1(T^*) = 1, W_{d+1}(T^*) = 0.5$)

		BS.	Obj.	Sub.	Inc_U	Inc_P	D.W.C	Eff.
	Benchmark	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	stock price = 12	6.241	30.670	31.659	3.4e-03	0.107	-0.989	3.6e-03
(a)	stock price = 10	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	stock price = 6	1.895	11.518	11.560	1.1e-02	0.274	-0.042	1.1e-02
	index price = 10	4.662	24.227	24.848	5.6e-03	0.147	-0.621	5.7e-03
(b)	index price = 6	4.662	24.227	24.848	5.6e-03	0.147	-0.621	5.7e-03
	index price = 2	4.662	24.227	24.848	5.6e-03	0.147	-0.621	5.7e-03
	correlation = 0.9	4.662	24.276	24.198	2.0e-03	0.133	0.079	2.0e-03
(c)	correlation = 0.6	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	correlation = 0.3	4.662	24.196	25.146	8.7e-03	0.138	-0.949	9.0e-03
	stock drift = 0.2	4.662	36.060	36.391	1.1e-03	0.124	-0.332	1.1e-03
(d)	stock drift = 0.15	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	stock drift = 0.12	4.662	18.747	19.848	-9.1e-06	0.147	-1.101	-9.7e-06
	index drift = 0.12	4.662	24.160	25.728	5.3e-03	0.139	-1.568	5.6e-03
(e)	index drift = 0.08	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	index drift = 0.06	4.662	24.260	24.602	8.0e-03	0.134	-0.342	8.1e-03
	risk-free rate = 0.06	4.944	21.232	21.813	4.2e-03	0.139	-0.581	4.3e-03
(f)	risk-free rate = 0.04	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	risk-free rate = 0.02	4.379	27.626	28.268	3.6e-03	0.135	-0.642	3.7e-03
	investment horizon = 7	5.459	28.141	29.451	4.3e-03	0.130	-1.310	4.5e-03
(g)	investment horizon = 5	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	investment horizon = 3	3.612	20.556	20.383	3.4e-03	0.143	0.173	3.4e-03
	stock volatility = 0.9	6.991	18.217	30.160	3.2e-02	0.161	-11.943	6.1e-02
(h)	stock volatility = 0.5	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	stock volatility = 0.4	3.970	23.315	23.308	1.7e-03	0.134	0.007	1.7e-03
	index volatility = 0.4	4.662	24.245	24.717	6.1e-03	0.136	-0.472	6.2e-03
(i)	index volatility = 0.3	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	index volatility = 0.1	4.662	24.158	27.564	3.9e-03	0.140	-3.406	4.5e-03
	strike = 12	4.147	22.660	23.261	5.0e-03	0.146	-0.601	5.1e-03
(j)	strike = 10	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	strike = 8	5.287	25.826	26.396	5.1e-03	0.129	-0.569	5.2e-03
	stock vesting period = 1.5	4.816	24.227	25.093	0.0e+00	0.000	-0.866	0.0e+00
(k)	stock vesting period = 1	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	stock vesting period = 0.5	4.662	24.227	24.616	7.2e-03	0.136	-0.389	7.3e-03
	option granted =2000 shares	4.662	24.227	38.112	-3.3e-01	0.117	-13.885	-5.9e-01
(l)	option granted = 200 shares	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	option granted = 20 shares	4.662	24.227	23.873	1.3e-03	0.133	0.354	1.3e-03
	stock granted = 2000 shares	4.662	24.227	25.027	4.8e-04	0.133	-0.800	4.9e-04
(m)	stock granted = 200 shares	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	Stock granted = 20 shares	4.662	24.227	25.799	1.1e-02	0.148	-1.572	1.2e-02
	cash endowment = 10000	4.662	24.227	23.895	1.9e-03	0.132	0.332	1.9e-03
(n)	cash endowment = 1000	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	cash endowment = 500	4.662	24.227	25.708	4.7e-03	0.136	-1.481	5.0e-03
	info disclosure time = 6.4 yr	4.662	14.644	14.481	4.3e-03	0.158	0.163	4.2e-03
(o)	info disclosure time = 6 yr	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	info disclosure time = 5.6 yr	4.662	63.244	69.205	4.5e-03	0.118	-5.961	5.0e-03
	information= 1.5	4.662	43.564	44.003	3.1e-03	0.122	-0.439	3.1e-03
(p)	information = 1	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	information = 0.2	4.662	7.632	8.137	2.1e-03	0.189	-0.505	2.2e-03
	noise = 0.8	4.662	24.227	24.671	4.7e-03	0.136	-0.444	4.8e-03
(q)	noise = 0.5	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	noise = 0.3	4.662	24.227	24.961	3.9e-03	0.136	-0.734	4.0e-03
	info quality = 0.9	4.662	24.227	24.813	5.9e-03	0.132	-0.586	6.0e-03
(r)	info quality = 0.6	4.662	24.227	24.848	2.5e-03	0.139	-0.621	2.6e-03
	info quality = 0.2	4.662	24.227	26.832	5.9e-03	0.142	-2.605	6.5e-03

Panel C: Insider Executives' Incentive of ESO with insider information as bad news ($W_1(T^*) = -1, W_{d+1}(T^*) = -0.5$)

		BS.	Obj.	Sub.	Inc_U	Inc_P	D.W.C	Eff.
	Benchmark	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	stock price = 12	6.241	0.778	1.051	9.1e-03	0.265	-0.273	1.3e-02
(a)	stock price = 10	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	stock price = 6	1.895	0.051	0.063	1.6e-02	0.857	-0.012	2.0e-02
	index price = 10	4.662	0.427	0.566	2.9e-02	0.393	-0.139	4.1e-00
(b)	index price = 6	4.662	0.427	0.566	2.9e-02	0.393	-0.139	4.1e-02
	index price = 2	4.662	0.427	0.598	2.9e-02	0.393	-0.139	4.1e-02
	correlation = 0.9	4.662	0.432	0.427	3.6e-03	0.345	0.005	3.6e-03
(c)	correlation = 0.6	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	correlation = 0.3	4.662	0.425	0.680	1.1e-02	0.374	-0.255	2.0e-02
	stock drift = 0.2	4.662	1.159	1.353	1.5e-02	0.304	-0.195	1.7e-02
(d)	stock drift = 0.15	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	stock drift = 0.12	4.662	0.215	0.314	2.4e-02	0.399	-0.100	3.9e-02
	index drift = 0.12	4.662	0.424	0.653	1.2e-03	0.370	-0.230	2.1e-03
(e)	index drift = 0.08	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	index drift = 0.06	4.662	0.430	0.532	1.5e-02	0.360	-0.102	1.9e-02
	risk-free rate = 0.06	4.944	0.356	0.466	1.6e-02	0.366	-0.111	2.2e-02
(f)	risk-free rate = 0.04	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	risk-free rate = 0.02	4.379	0.512	0.682	6.7e-03	0.362	-0.170	9.3e-03
	investment horizon = 7	5.459	0.941	1.182	1.4e-02	0.276	-0.241	1.9e-02
(g)	investment horizon = 5	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	investment horizon = 3	3.612	0.094	0.138	1.2e-02	0.576	-0.043	1.8e-02
	stock volatility = 0.9	6.991	0.014	0.036	1.6e-02	0.528	-0.022	7.9e-02
(h)	stock volatility = 0.5	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	stock volatility = 0.4	3.970	0.946	1.085	1.4e-02	0.362	-0.139	1.6e-02
	index volatility = 0.4	4.662	0.428	0.547	2.0e-02	0.349	-0.119	2.7e-02
(i)	index volatility = 0.3	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	index volatility = 0.1	4.662	0.421	0.798	1.4e-02	0.374	-0.377	3.4e-02
	strike = 12	4.147	0.262	0.347	1.4e-02	0.407	-0.085	1.9e-02
(j)	strike = 10	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	strike = 8	5.287	0.706	0.934	7.1e-03	0.326	-0.228	9.8e-03
	stock vesting period = 1.5	4.816	0.427	0.598	0.0e+00	0.000	-0.171	0.0e+00
(k)	stock vesting period = 1	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	stock vesting period = 0.5	4.662	0.427	0.537	5.8e-03	0.359	-0.110	7.5e-03
	option granted =2000 shares	4.662	0.427	1.207	-4.7e+00	0.352	-0.780	-2.9e+01
(l)	option granted = 200 shares	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	option granted = 20 shares	4.662	0.427	0.496	3.4e-03	0.355	-0.068	3.9e-03
	stock granted = 2000 shares	4.662	0.427	0.602	1.4e-03	0.343	-0.175	2.2e-03
(m)	stock granted = 200 shares	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	Stock granted = 20 shares	4.662	0.427	0.571	4.1e-02	0.380	-0.144	5.8e-02
	cash endowment = 10000	4.662	0.427	0.438	1.7e-03	0.350	-0.010	1.7e-03
(n)	cash endowment = 1000	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	cash endowment = 500	4.662	0.427	0.626	2.4e-02	0.358	-0.199	3.8e-02
	info disclosure time = 6.4 yr	4.662	0.159	0.167	1.6e-02	0.506	-0.008	1.7e-02
(o)	info disclosure time = 6 yr	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	info disclosure time = 5.6 yr	4.662	2.036	3.365	1.4e-02	0.217	-1.329	2.8e-02
	information= -1.5	4.662	0.063	0.092	-2.4e-03	0.500	-0.029	-3.9e-03
(p)	information = -1	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	information = -0.2	4.662	3.561	4.170	1.2e-02	0.237	-0.609	1.4e-02
	noise = -0.8	4.662	0.427	0.587	5.0e-03	0.359	-0.160	7.3e-03
(q)	noise = -0.5	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	noise = -0.3	4.662	0.427	0.551	1.7e-02	0.362	-0.124	2.3e-02
	info quality = 0.9	4.662	0.427	0.682	5.8e-03	0.366	-0.255	1.1e-02
(r)	info quality = 0.6	4.662	0.427	0.566	1.8e-02	0.368	-0.139	2.4e-02
	info quality = 0.2	4.662	0.427	0.504	7.7e-03	0.360	-0.077	9.2e-03

Table 2 lists the results of ESO incentive sensitivity analysis, namely, how does a parameter change affect the change of the ESO incentive for a high-volatility regime. Panel A reports the results of outsider executives. Panel B (C) reports the results of insider executives who acknowledge a noisy information as good (bad) news. Each panel displays all the determinants affecting (European) ESO efficiency [Eff.], which is defined as the deadweight-cost-adjusted incentive. The deadweight cost [D.W.C.] is the objective price [Obj.] net of the subjective price [Sub.]. The incentive [Inc_U] is the percentage change of outsider executives' derived utility w.r.t. stock spot price. We also list a price incentive [Inc_P], which is the percentage change of executives' logarithm subjective ESO price w.r.t. stock spot price change. Each subpanel lists three levels (low, benchmark, high *w.r.t.* the determinant) of results from top to bottom.

Appendix

Proof of Proposition 4.

i. If the firm's stock S_1 is the only non-tradable asset in the blackout period, then $\forall t \in [t_{\mathfrak{B}}, T]$ and $\forall u \in [t, T]$, $K(u,\omega) = [\phi_1(u), \phi_1(u)] \times (-\infty, \infty)^{d-1}$, where $\phi_1(u) = \frac{[N^S(t_{\mathfrak{B}})+N(t_{\mathfrak{B}})\Phi(u)]S_1(u)}{X(u)}$; $N^S(t_{\mathfrak{B}})$ and $N(t_{\mathfrak{B}})$ are the number of shares of the firm's stock and the number of ESOs that the insider executive has when blackout starts, *i.e.*, at $t_{\mathfrak{B}}$; $\Phi(u)$ is the replication position for the ESO. Note that $N^S(t_{\mathfrak{B}})$ and $N(t_{\mathfrak{B}})$ are constant over $[t_{\mathfrak{B}}, T]$ due to the non-tradability, but $\Phi(u)$ is time varying.

In this case, $\widetilde{K} = (-\infty, \infty) \times \{0\}^{d-1}$ and the scalar support function $\delta(v)$ determining $v^{\mathbb{G}}(u)$ and $v^{\mathbb{F}}(u)$ is $\delta(v) = -\phi_1(u) \times v_1(u)$. Then,

$$v^{\mathbb{G}}(u) \equiv \underset{v\in\widetilde{K}}{\operatorname{argmin}}[2\delta(v) + \|\sigma^{-1}(u)[b(u) + v(u) - r(u)\mathbf{1}] + a(u)\|^2];$$

$$v^{\mathbb{F}}(u) \equiv \underset{v\in\widetilde{K}}{\operatorname{argmin}}[2\delta(v) + \|\sigma^{-1}(u)[b(u) + v(u) - r(u)\mathbf{1}]\|^2].$$

From $\widetilde{K}$, we see that $v_i^{\mathbb{G}}(t) = v_i^{\mathbb{F}}(t) = 0$ for $i = 2, \dots, d$, and we also see that $v_1^{\mathbb{G}}(t) < 0$ and $v_1^{\mathbb{F}}(t) < 0$ are allowed. From Eq. (15) and Eq. (16), it is not hard to see that

$$v_1^{\mathbb{G}}(u) = \frac{1}{h_{1,1}(u)}\left(\phi_1(t,u) - \textstyle\sum_{i=1}^{d}(b_i(u) - r(u))h_{i,1}(u) - g_{11}(u)a_1(u)\right),$$

$$v_1^{\mathbb{F}}(u) = \frac{1}{h_{1,1}(u)}\left(\phi_1(t,u) - \textstyle\sum_{i=1}^{d}(b_i(u) - r(u))h_{i,1}(u)\right).$$

This implies that $v_1^{\mathbb{F}}(u) - v_1^{\mathbb{G}}(u) = \frac{g_{11}(u)}{h_{1,1}(u)}a_1(u)$. As discussed in the paragraph before Eq.(18), for log utility, the insiders' optimal constrained portfolio process is given by

$$\pi^{\mathbb{G}}(u) = [\sigma^{\top}(u)]^{-1}\left(\theta(u) + \sigma^{-1}(u)v^{\mathbb{G}}(u) + a(u)\right) = [\sigma^{\top}(u)]^{-1}\left(\theta(u) + a(u)\right) + h(u)v^{\mathbb{G}}(u),$$

while

$$\pi^{\mathbb{F}}(u) = [\sigma^{\top}(u)]^{-1}\left(\theta(u) + \sigma^{-1}(u)v^{\mathbb{F}}(u)\right) = [\sigma^{\top}(u)]^{-1}\theta(u) + h(u)v^{\mathbb{F}}(u).$$

From Eq. (2) it is clear that if $\sigma_{i,1}(u) = 0$ for all $i = 2, \dots, d$, then $a(u)$ is independent from $S_i(u) = 0$ for all $i = 2, \dots, d$. Thus, $[\sigma^{\top}(u)]^{-1}a(u) = (g_{11}(u)a_1(u), 0 \dots, 0)^{\top}$. So, $\pi^{\mathbb{G}}(u) = \pi^{\mathbb{F}}(u)$, and $\Delta\mathcal{J}(x_t, t, T) = 0$.

ii. If $\exists\, i = 2, \dots, d, s.t. \sigma_{i,1}(u) \neq 0$, then $\pi^{\mathbb{G}}(u)$ is still a function of $a(u)$, and the rest of the proof is similar to Proposition 1.

iii. If $K(u,\omega) = [\phi_1(t,u), \phi_1(t,u)] \times \dots \times [\phi_d(t,u), \phi_d(t,u)]$, then $\pi^{\mathbb{G}}(u) = \pi^{\mathbb{F}}(u)$, and $\Delta\mathcal{J}(x_t, t, T) = 0$, as required. □

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